18/12/24

1 - Laminar & Turbulent Flow in Pipes

<u>Laminar Flow</u>: In The laminar flow, the fluid particles move along straight paths in layers such that path of each individual fluid particle do not cross their neighbouring particles.

* Laminar flow is possible @ low velocities when the fluid is highly viscous, when velocity ↑es or less viscous of fluid particles do not move in straight paths.
* The fluid particles move in random manner resulting in general mixing of particles. This type of flow is known as turbulent f

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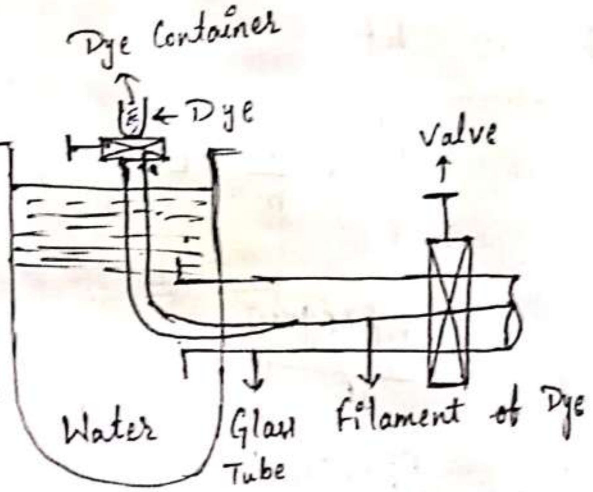
<u>Reynold's Exp</u>:



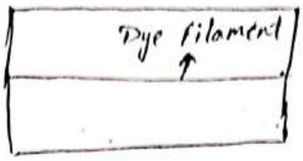
A laminar flow changes to turbulent flow when velocity is increased or dia of pipe ↑ed or viscosity of fluid ↓es.

* O. Reynold was 1st scientist to demonstrate transition form from laminar to turbulent not only on the mean velocity but on the quantity "$\frac{\rho V D}{\mu}$".
* This quantity is a dimensionless quantity & it is called as 'Reynold's number'.
* The type of flow is determined from the $\frac{\rho V D}{\mu}$ is was demonstrated by O. Reynold in 1883.

→ The water from the tank was allowed to flow through the glass tube then a liquid dye having same sp. wt as water was introduced into the glass tube as shown in fig.

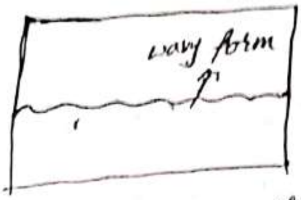
The following observations were made by O. Reynold is :

→ When the velocity of flow was low the dye filament in the glass tube is in the form of a Straight line. This st. line was parallel to glass tube which in a case of laminar flow as shown in fig (a).



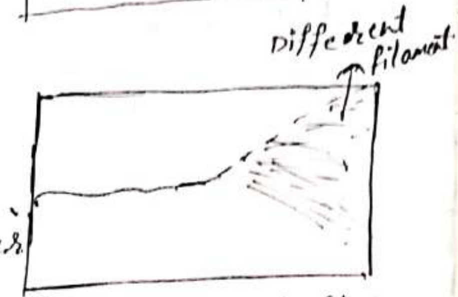
(a) Laminar Flow.

→ With the Ier of velocity of flow, the dye filament was no longer a straight line, it becomes wavy as shown in fig (b). The flow is no longer a laminar.



Transition (b) ~~Turbulent~~ Flow

→ With further increase of velocity, the wavy dye filament broke up & finally diffused. This means the fluid particles of the dye at higher velocity are moving in random manner which shows the case of turbulent flow.



(c) Turbulent Flow

→ Thus in the case of turbulent flow, mixing of water is flowing & disorderly.

→ In case of laminar flow, the loss of pressure head was found to be proportional to velocity but in case of turbulent flow, loss of head is exactly $\boxed{h_f \propto V^n}$ where n varies from 1.75 to 2.

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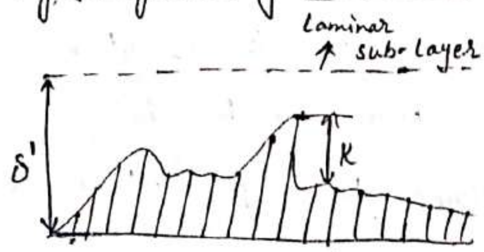
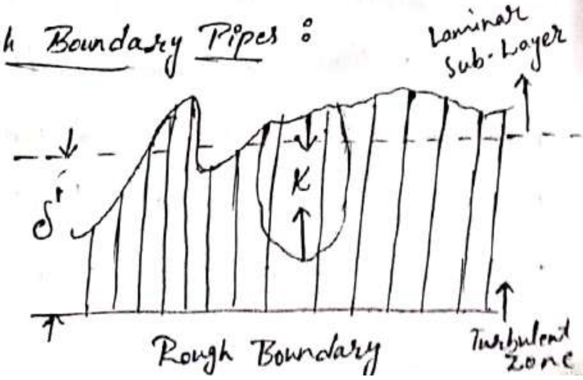
Hydrodynamically Smooth & Rough Boundary Pipes :



Smooth Boundary



Rough Boundary

Let K = Avg. Height of the irregularities projecting from the surface of boundary as shown in fig. If the value of K is large then the boundary is rough. If the value of K less than the boundary is smooth boundary. The classification of rough & smooth boundary depends on boundary characteristics, proper characteristics of the flow also to be considered.

$K \propto \delta'$

$K < \delta'$ ⇒ Smooth Boundary

$K > \delta'$ ⇒ Rough Boundary.

If $K/\delta' < 0.25$ ⇒ Smooth Boundary

$K/\delta' > 6$ ⇒ Rough Boundary

$0.25 < K/\delta' < 6$ ⇒ Then it is having transition.

In terms of roughness, reynold's no $= \frac{uK}{\nu}$.

If $\frac{uK}{\nu} < 4$ ⇒ Smooth Boundary

$\frac{uK}{\nu}$ lies b/w 4 & 100 ⇒ then it is ~~Rough Boundary~~ Transition.

$\frac{uK}{\nu} > 100$ ⇒ Rough Boundary

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Velocity Distribution for Turbulent Flow in Smooth Pipes:

The velocity distribution in smooth or rough pipes is given by equation $\boxed{u = \frac{u^*}{K} \log_e y + c}$ → ①. It may be seen that at $y = 0$, the velocity u is ∞. This means the velocity u is +ve at some distance far away from the wall & then it becomes negative. Hence at some finite distance from wall the velocity will be zero. ∴ Let the distance from pipe valve is y'. Now the constant C is determined from the boundary condition i.e., at $y = y'$ & $u = 0$. Hence the equation① becomes

$$0 = \frac{u^*}{K} \log_e y' + c$$

$$\boxed{c = -\frac{u^*}{K} \log_e y'}$$

Sub c in the eq ①

$$u = \frac{u^*}{K} \log_e y - \frac{u^*}{K} \log_e y'$$

$$u = \frac{u^*}{K} [\log_e y - \log_e y']$$

$$u = \frac{u^*}{K} \left[\log \frac{y}{y'}\right]$$

If the value of K is 0.4 then $u = \frac{u^*}{0.4}\left(\log \frac{y}{y'}\right)$

$$\therefore u = 2.5\, u^* \log \frac{y}{y'}$$

$$\log \frac{y}{y'} = 2.3 \log_{10} (y/y')$$

$$u/u^* = 2.5 \times 2.3 \log_{10} (y/y')$$

$$u/u^* = 5.75 \log_{10} (y/y')$$

For smooth boundary, there exist a laminar sub-layer as shown in fig. The velocity distribution in laminar sub-layer is parabolic. In the laminar sub layer log velocity distribution holds good. Thus it can be assumed that y' assumed to δ' where.

δ' = thickness of laminar sub-layer.

$$\therefore y' \propto \delta'$$

Then the value of y' is given as

$$y' = \frac{\delta'}{107} \; ; \; \delta' = \frac{11.6 \nu}{u^*}$$

ν = Kinematic viscosity

$$y' = \frac{11.6\nu}{u^*} \times \frac{1}{107} = \frac{0.108 \nu}{u^*}$$

Sub y' in above eq.

$$\frac{u}{u^*} = 5.75 \log_{10} y \left[\frac{0.108 \nu}{u^*}\right]$$

$$\frac{u}{u^*} = 5.75 \log_{10} \frac{u^* y}{\nu} + 5.55$$

$$\boxed{u = 5.75 \log_{10} \frac{u^* y}{0.108\nu} + 5.55}$$

Velocity Distribution for turbulent flow in rough pipes :

$$\frac{u}{u^*} = 5.75 \log_{10}\left[\frac{y}{\frac{K}{30}}\right]$$

$$= 5.75 \log_{10}\left(\frac{y}{K}\right) \times 30$$

$$= 5.75 \log_{10}\left(\frac{y}{K}\right) + 5.75 \log_{10}(30)$$

$$\frac{u}{u^*} = 5.75 \log_{10}\left(\frac{y}{K}\right) + 8.5$$

① A pipe line carrying water has the avg height of irregularities projecting from the surface of boundary of the pipe has 0.15 mm. What type of boundary is it. The shear stress developed is $4.9 N/m^2$, the kinematic viscosity of water is 0.01 strokes.

Sol: Given that :

Avg height of irregularities $K = 0.15 mm = 0.15 \times 10^{-3} m$

Shear stress developed $\tau_0 = 4.9 N/m^2$

1 stroke $= 1 cm^2/sec$

Kinematic viscosity ν : 0.01 strokes

$= 0.01 \times 10^{-4} m^2/sec$

Density of water $\rho = 1000 kg/m^3$

Shear velocity $u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 m/sec$

Roughness Reynold's Number $= \frac{u_* K}{\nu} = \frac{0.07 \times 0.15 \times 10^{-3}}{0.01 \times 10^{-4}}$

$= 10.5$

$\because \frac{u_* K}{\nu}$ lies in b/w 4 & 100, then the pipe surface behaves as transition.

1) A rough pipe is of dia 8cm, the velocity at a point 3cm from value is 30% more than the velocity at a point 1cm from pipe value determine the avg. height of roughness. $\boxed{K=?}$

Sol: Given data:

Dia of pipe = 8cm = 0.08 m

Let velocity of flow 1cm away from wall = u

velocity of flow 3cm away from wall = 1.3u

The velocity distribution for rough pipe is given by eq

$$\frac{u}{u^*} = 5.75 \log_{10}\left(\frac{y}{K}\right) + 8.5$$

For point 1 cm from wall, we have

$$\frac{u}{u^*} = 5.75 \log_{10}\left[\frac{1}{K}\right] + 8.5 \rightarrow (1)$$

For point 3cm from wall, we have

$$\frac{1.3u}{u^*} = 5.75 \log_{10}\left[\frac{3}{K}\right] + 8.5 \rightarrow (2)$$

Divide eq (2) by (1)

$$\frac{\frac{1.3\cancel{u}}{\cancel{u^*}}}{\frac{\cancel{u}}{\cancel{u^*}}} = \frac{5.75 \log_{10}(3/K) + 8.5}{5.75 \log_{10}(1/K) + 8.5}$$

$$1.3 = \frac{5.75 \log_{10}^{3} - \log_{10}^{K} + 8.5}{5.75 \log_{10}^{1} - \log_{10}^{K} + 8.5}$$

$$1.3 = \frac{5.75 \log_{10}^{3} - \log_{10}^{K} + 8.5}{5.75 \log_{10}^{1} - \log_{10}^{K} + 8.5}$$

$$7.475(\log_{10}^{1} - \log_{10}^{K}) + 11.05 = 5.75 \log_{10}^{3} - \log_{10}^{K} + 8.5$$

$$7.475(0 - \log_{10}^{K}) - 5.75(\log_{10}^{3} - \log_{10}^{K}) = 8.5 - 11.05$$

$$-7.475 \log_{10}^{K} - 5.75(0.477 - \log_{10}^{K}) = -2.55$$

$$-7.475 \log_{10}^{K} - 2.74 + 5.75 \log_{10}^{K} = -2.55$$

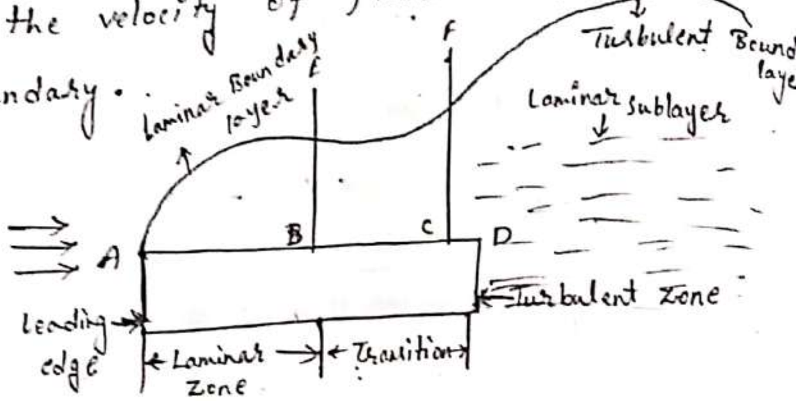
$$-1.725 \log_{10}^{K} = 0.1933$$

$$\log_{10}^{K} = \frac{0.1933}{1.725}$$

$$\boxed{K = 0.89\ cm}$$

Boundary Layer: This was first introduced by L. Prandtyl in 1904.

When a real fluid flow through a solid boundary a layer of the fluid which comes in contact to the boundary & sticks to it & the condition of no slip occurs when the velocity of fluid is equal to the velocity of boundary.



* Velocity of flowing fluid increases rapidly at the boundary surface & approaches max. velocity.
* The layer adjacent to the boundary is known as boundary layer.
* Boundary layer is formed when there is a relative motion b/w the fluid & the boundary layer.
* Fluid exerts shear force on the boundary layer & the boundary layer exerts equal & opposite shear resistance force on the fluid

Moody's Diagram } As in
stroke law } notes

Laminar Flow Around a Sphere - Strokes Law:

In 1815, G.G Strokes developed analytically an expression for the resistance F_D experienced by a sphere of diameter D, moving with a constant velocity V (i.e.., there is no acceleration) in a fluid of viscosity μ, which is given by

$$F_D = 3\pi\mu VD. \longrightarrow (1)$$

It is known as stroke's law which has been verified experimentally & is found to hold good for values of Reynold's no (VD/ν) less than 0.2.

If the ... through the fluid under its own weight at a constant velocity V, then the buoyant force plus the resistance to its motion must be equal to its weight.

Thus we obtain :

$$\left[\frac{\pi}{6} D^3\right] w + F_D = \left[\frac{\pi}{6} D^3\right] w_s$$

$$F_D = \left[\frac{\pi}{6} D^3\right] (w_s - w)$$

in which w is sp. wt of the fluid, w_s is the sp. wt of the sphere & $\left[\frac{\pi}{6} D^3\right] (w_s - w)$ represents the submerged weight of the sphere.

* Introducing the value of F_D from eq (1) in the above expression & solving for V, we obtain :

$$V = \frac{D^2}{18\mu} (w_s - w) \rightarrow (2)$$

The velocity V is given by eq (2) is called terminal fall velocity, which is defined as the velocity attained by a body in falling through a fluid at rest, when the drag on the body is equal to the submerged weight of the body.

* viscosity μ of a fluid can be determined experimentally. If in eq (2) the fall velocity 'V' of a sphere of known dia D, moving down through the fluid is measured.

* The drag F_D for any body moving with velocity V in a fluid of mass density ρ may be expressed as

$$F_D = C_D A \frac{\rho V^2}{2}$$

in which C_D = Coefficient of drag

A = The projected area of the body ⊥ to the direction of flo

for the sphere ; $F_D = 3\pi\mu V D$

$$A = \frac{\pi D^2}{}$$

Thus by substituting for F_D & A in the ... we get

$$C_D = 24\left[\frac{\mu}{\rho V D}\right] = \frac{24}{Re} \rightarrow (3)$$

Eq (3) represents the drag coefficient for sphere on the basis of Stroke's law.

Exp's have shown that eq (3) ~~holds~~ holds good for reynolds no < 0.2 provided the sphere is moving in an infinite fluid.

However, if the fluid through which the sphere moves is not infinite in extent but is confined within a container of finite dimensions, then the resistance to motion is increased & insuch a case the drag coefficient is given by the following modified expression,

$$C_D' = \frac{24}{Re}\left[1+2.1\frac{D}{D_1}\right]$$ where D_1 = Smallest lateral dimension of container & D is the dia of the sphere.

<u>Laminar Boundary Layer</u>: Consider a stationary plate & hence velocity of the fluid on the plate is zero.

* After some time, from the boundary layer, fluid attains some velocity.
* Boundary layer starts from leading edge.
* Near the leading edge where the flow thickness is small, the flow in the boundary layer is laminar. this layer is known as laminar boundary layer.
* The length upto which the laminar boundary layer exist is known as laminar zone. The distance is equal to 5×10^5 for a plate according to reynold's no.

$$(Re)_x = \frac{U\times x}{\nu} \Longrightarrow 5\times10^5 = \frac{U\times x}{\nu}$$

ν = Kinematic Viscosity

x = Distance from leading edge.

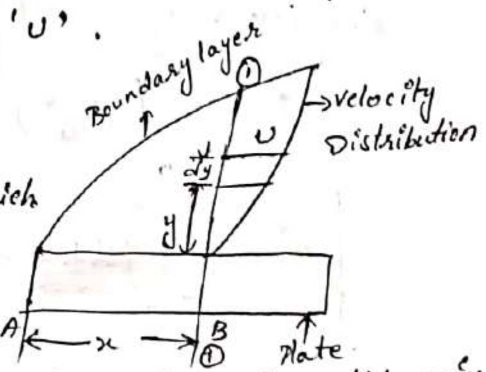
Turbulent Boundary Layer :

* If the length of the plate is more than the distance x from the above equation the thickness of the boundary layer will go on increasing.
* The flow is distributed & irregular which leads to transition from laminar to turbulent boundary layer.
* The zone in which laminar changes to turbulent boundary laye
* After transition zone, thickness of layer increases known as turbulent zone.

Boundary Layer Thickness (δ) :

* Distance from the boundary of solid body measured in y-direction to the point where the velocity of fluid is approximately equal to 0.99 times the free stream velocity 'U'.

Displacement Thickness (δ^*) :



* The distance ⊥ to the boundary by which the free stream is displaced due to the formation of the boundary layer.
* Consider the flow velocity equal to U over a plate, at a distance 'x' from leading section.
* Consider section 1-1 @ point B, the velocity is zero & at C velocity is U. The distance b/w B & C is δ.
* Let y = Distance of elemental strip from the plate

  dy = Thickness of elemental strip

  u = velocity of fluid at elemental strip

  b = width of the ~~fluid~~ plate

Area of the elemental strip, $dA = b \times dy$

Mass per second flowing through the strip $= \rho \times u \times$ Area of strip

$= \rho \times u \times b \times dy \rightarrow ①$

Mass per second flowing through the strip $= \rho \times U \times b \times dy \longrightarrow ②$

If U is more than u, reduction in mass per second flowing through the strip is

$= \rho \times U \times b \times dy - \rho \times V \times b \times dy$

$= \rho \times b \times dy\ (U - V) \rightarrow (3)$

$\therefore$ Total reduction in mass of fluids through BC due to plate $= \int \rho \times b \times dy\ (U - V)$

$= \rho \times b \int (U - V)\, dy$

If δ^* is the distance displaced then loss of mass of fluid per second flowing through the distance is $\rho \times V \times A$

$= \rho \times U \times \delta^* \times A \rightarrow (4)$

Equating the above eqs

$$\rho \times \cancel{b} \int_0^{\delta} (u - v)\, dy = \cancel{\rho} \times U \times \delta^* \times \cancel{b} \times dy \qquad [\because A = b \times dy]$$

$$\delta^* = \int_0^{\delta} \left(\frac{u - v}{u}\right) dy$$

$$\boxed{\delta^* = \int_0^{\infty} \left(1 - \frac{v}{u}\right) dy}$$

<u>Momentum</u> <u>Thickness (θ)</u>: Distance measured $\perp$ to the boundary of the solid body by which the boundary should be compenate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by θ.

Momentum of the fluid $=$ mass $\times$ velocity

$= (\rho \times u \times b \times dy) \times u$

$= \rho \times b \times dy \times u^2 \rightarrow (1)$

Momentum of the fluid in the absence of boundary layer

$= (\rho \times u \times b \times dy)\ U \rightarrow (2)$

Loss of momentum through elemental strip

$= eq (2) - eq (1)$

$= (\rho \times u \times b \times dy)\ U - (\rho \times b \times dy \times u^2)$

$= \rho \times b \times dy \times u\ (U - u) \rightarrow (3)$

Total loss of momentum per second,

$$\int_0^{\delta} \rho \times b \times dy \times u\,(U-u) \longrightarrow (4)$$

Let θ = distance by which plate is displaced when the fluid is flowing with a constant velocity U.

Mass of fluid through $\times\ \theta \times$ velocity

$= \rho \times$ Area $\times$ velocity $\times\ \theta \times$ velocity

$= (\rho \times \theta \times b \times U)\, U$

$= \rho \times \theta \times b \times U^2 \longrightarrow (5)$

Equate (4) & (5)

$$\int_0^{\delta} \rho \times b \times dy \times u\,(U-u) = \rho \times \theta \times b \times U^2$$

$$\theta = \int_0^{\delta} \frac{1}{U}\left(u - \frac{u^2}{U}\right) dy$$

$$\boxed{\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$

Energy Thickness (δ^{**}) : To reduce kinetic energy distance measured $\perp$ to the boundary of the solid body by which boundary should be displaced to compensate the reduction in kinetic energy of the flowing fluid on accound of boundary layer formation & is denoted by δ^{**}

Kinetic energy of fluid;

$= \frac{1}{2} mv^2 = \frac{1}{2}(\rho \times u \times b \times dy)\,u^2 \longrightarrow (1)$

Kinetic energy of fluid in absence of boundary layer

$= \frac{1}{2}(\rho \times u \times b \times dy)\,U^2 \longrightarrow (2)$

Loss of kinetic energy through strip,

$=$ eq (2) $-$ eq (1)

$= \frac{1}{2}(\rho \times u \times b \times dy)\,U^2 - \frac{1}{2}(\rho \times u \times b \times dy)\,u^2$

$= \frac{1}{2}(\rho \times u \times b \times dy)\,(U^2 - u^2) \longrightarrow (3)$

Total loss of Kinetic energy of fluid through BC in fig

$$= \int_0^{\delta} \frac{1}{2}(\rho \times u \times b)(U^2 - u^2)\,dy$$

$$= \frac{1}{2} \times \rho \times b \int_0^{\delta} u(U^2 - u^2)\,dy \rightarrow (4)$$

Let δ^{**} = Distance by which the plate is displaced to compensate for the reduction of Kinetic energy.

$$\frac{1}{2}(\rho \times b \times \delta^{**} \times U)\,U^2 = \frac{1}{2} \times \rho \times b \times \delta^{**} \times U^3 \rightarrow (5)$$

$$eq\ (4) = eq\ (5)$$

$$\frac{1}{2}\rho \times b \int_0^{\delta} u(U^2 - u^2)\,dy = \frac{1}{2} \times \rho \times b \times \delta^{**} \times U^3$$

$$\int_0^{\delta} u(U^2 - u^2)\,dy = \delta^{**} U^3$$

$$\boxed{\delta^{**} = \frac{u}{U}\int_0^{\delta}\left(1 - \frac{u^2}{U^2}\right)dy}$$

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2. Uniform Flow in Open Channels

Hydraulics : It is an engg. science which deals with the behaviour of water, especially the conveyance of water in pipe & channels.

Channel : A channel is a narrow passage through which water is transported from one place to another or from one source to another.

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Pipe Flow	Channel Flow
* A pipe flow is a type of liquid flow within a closed conduit.	* The flow is open to the atmosphere.
* It doesn't exert direct atmospheric pressure as being confined within closed conduit.	* It exert directly atmospheric pressure.

5M

Types of Channels :

* Channels are classified into 3 types.

a) Based on Cross-Sectional Area : 1) Natural Channels
2) Artificial "

1) **Natural :** These channels have irregular c/s.
* These are developed in natural ways.
Ex: Rivers, Canals, lakes, streams etc..,

2) **Artificial :** These channels are man-made channels.
* These have regular shape.
Ex: Rectangular, Triangular, Trapezoidal, Circular.

1) Based on shape of Cross section:

1) **Prismatic channels:** A channel is said to be prismatic whose bed slope & cross-section is constant.

Ex: All artificial channels are prismatic channels.

2) **Non-Prismatic:** A channel is said to be non-prismatic whose c/s & bed slope is not constant.

Ex: All Natural channels & ' .

c) **Based on Boundaries:**

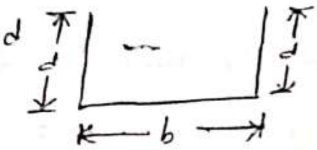
1) Rigid Boundary: 2) Non-Rigid or flexible Boundary.

Geometric Parameters of a Channel:

Depth (d): It is the vertical distance b/w surface of water to the bottom of the channel.

Top width (T): It is the width of a free surface of liquid.

Wetted Perimeter (P): It is the length of the channel boundary in contact with the flowing water at any section. $P = d + b + d = b + 2d$



Wetted Area or C/s Area:

It is the c/s area of the flow section of the channel.

Hydraulic Radius / Hydraulic Mean Depth: It is the ratio of wetted area to the wetted perimeter.

$$R \text{ or } m = \frac{A}{P}$$

Hydraulic Depth (D): It is the ratio of wetted area to the top width. $D = \frac{A}{T}$

Section Factor : It is the product of wetted area to square root of hydraulic depth.

$$Z = A \times \sqrt{D}$$

$$Z = A \times \sqrt{\frac{A}{T}}$$

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Velocity Distribution in a Channel section :

At a distance of 0.05z to 0.05 m from free surface, velocity will be maximum.

From various experiments we get average velocity is found at a distance of ~~point~~ 0.6m from free surface.

$\therefore$ Exact Avg. Velocity $= \dfrac{V_{0.2m} + V_{0.8m}}{2}$

Flow Analysis :

Discharge through open channel by Chezy's Formula :

Weight of the liquid = Specific Weight $\times$ Volume

$= w \times L \times b \times b$ $\quad (b \times b = A)$

$= w \times L \times A$

Frictional Resistance $= f \times$ Surface Area $\times (\text{velocity})^n$

$\because n = 2$ $\quad = f \times P \times L \times V^2$

Surface Area = Perimeter $\times$ length

~~Frictional Resistance = $f \times P \times L \times V^2$~~

Forces acting in the direction of flow are :

① Self Weight in the direction of flow $= w \sin\theta$

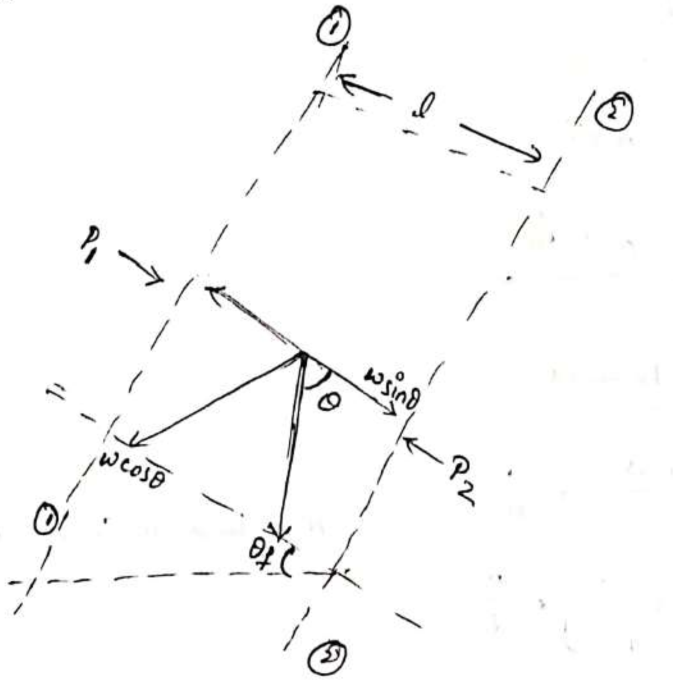
② Frictional Resistance

③ Force acting at section ①-① $= P_1$

④ Force acting at section ②-② $= P_2$

Resolving all the forces in the direction of x.

$P_1 + w\sin\theta - f\,PLV^2 - P_2 = 0$



As per equilibrium conditions, forces acting at section 1 & 2 are same. $P_1 = P_2$

$$\cancel{P_2} + w\sin\theta - fPLV^2 - \cancel{P_2} = 0$$

$$w\sin\theta = fPLV^2$$

$$V^2 = \frac{w\sin\theta}{fP \times L}$$

$$= \frac{(w \times A \times \cancel{L})\sin\theta}{f \times P \times \cancel{L}}$$

$$= \frac{(wA)\sin\theta}{fP}$$

$$V = \sqrt{\frac{w \times A \times \sin\theta}{f \times P}}$$

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P}} \times \sqrt{\sin\theta}$$

$$V = \sqrt{\frac{w}{f} \times m \times \theta} \qquad \because \sqrt{\frac{w}{f}} = C$$

$$\boxed{V = C\sqrt{m\theta}}$$

or

$$\boxed{V = C\sqrt{mi}}$$

Where : V = velocity of flow

C = Chezy's constant

m = Hydraulic Mean depth

i = Bed slope.

$\therefore$ Discharge $Q = A \times V$

$$\boxed{\therefore Q = A \times C\sqrt{mi}}$$

Ganguillet - Kutter formula:

$$C = \frac{23 + \frac{0.00155}{i} + \frac{1}{N}}{1 + \left[23 + \frac{0.00155}{9}\right] \times \frac{N}{\sqrt{m}}}$$; N = Roughness Coefficient.

Bazin's formula:

$$C = \frac{157.6}{181 f \times \frac{K}{\sqrt{m}}}$$; K = Bazin constant.

Manning's formula:

$$C = \frac{1}{N} m^{1/6}$$

$$V = C\sqrt{mi}$$

$$\boxed{\therefore V = \frac{1}{N} m^{2/3} i^{1/2}}$$

(1) Find the velocity of flow & rate of flow of water through the rectangular channel of 6m wide & 3m deep. When it is running full. The channel is having bed slope as 1:2000. Take chezys constant C = 55.

Sol: Given data:

Width of a channel = 6m

Depth of a channel = 3m

Chezy's Constant C = 55

Bed slope $i = 1:2000 = \frac{1}{2000}$

Perimeter is the length of sides which are in contact with fluid containing in the channel.

Velocity $V = C\sqrt{mi}$

$$m = \frac{A}{P} = \frac{b \times d}{b + 2d} = \frac{6 \times 3}{6 + 2(3)} = 1.5\,m$$

$$V = 55\sqrt{1.5 \times \frac{1}{2000}} = 1.506\,m/sec.$$

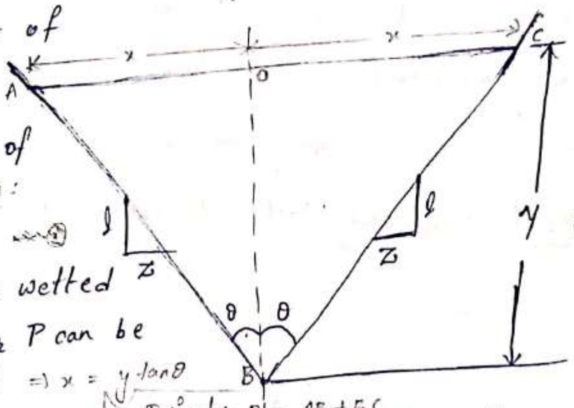
Discharge, $Q = A \times V$

$$= 6 \times 3 \times 1.506$$
$$= 27.108\ m^3/sec$$

③ Find the bed slope of a rectangular channel of width 8m when the depth of water is 2m & the rate of flow is $80m^3/sec$, take $C = 50$.

~~Velocity Distribution in a Channel Section:~~

Most Efficient Triangular Channel Section: For a triangular channel section, if θ is the angle of inclination of each of the slopping sides with the vertical & y is the depth of flow.



From the Area of A:

$A = 2\left[\frac{1}{2} \times x \times y\right] = xy$ ⓐ

* The following expression for the wetted area A & the wetted perimeter P can be written:

From ΔABO: $\tan\theta = \frac{x}{y} \Rightarrow x = y\tan\theta$

$\therefore A = y\tan\theta \cdot y$

$$A = y^2 \tan\theta$$

$$y = \sqrt{A/\tan\theta} \rightarrow ①$$

$$P = (2y)\sec\theta \rightarrow ②$$

Perimeter, $P = AB + BC$

From ΔABO: $\cos\theta = \frac{BO}{AB} = \frac{y}{AB}$

$AB = \frac{y}{\cos\theta} = y\sec\theta$

$BC = \frac{y}{\cos\theta} = y\sec\theta$

$P = AB + BC = 2y\sec\theta$

Substituting the value of y from eq ① & ②

$$P = 2\left[\sqrt{\frac{A}{\tan\theta}}\right]\sec\theta \Rightarrow P = 2\sqrt{\frac{A}{\tan\theta}}\sec\theta \rightarrow ②$$

$$P = \frac{2\sqrt{A}}{\sqrt{\tan\theta}}(\sec\theta) \rightarrow ③$$

for maximum perimeter, $\frac{dP}{d\theta} = 0$

Assuming the Area A to be constant, eq ③ can be differentiated w.r.t θ & equated to zero for obtaining the condition for min P.

$$\frac{dP}{d\theta} = \frac{d}{d\theta}2\sqrt{A}\frac{\sec\theta}{\sqrt{\tan\theta}} = 2\sqrt{A}\frac{d}{d\theta}\frac{\sec\theta}{\sqrt{\tan\theta}}$$

Thus $$\frac{dP}{d\theta} = 2\sqrt{A}\left[\frac{\sec\theta\tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{3/2}}\right] = 0$$

or $$\sec\theta\,(2\tan^2\theta - \sec^2\theta) = 0$$

$$\because \sec\theta \neq 0$$

$$2\tan^2\theta - \sec^2\theta = 0$$

$$\sqrt{2}\tan\theta = \sec\theta$$

$$\boxed{\theta = 45^\circ \text{ or } Z = 1} \rightarrow ④$$

Discharge Minimum &
Perimeter Minimum } Condition which so that a channel is efficient
↓
Minimum lining
↓
Minimum Cost

* reduces the perimeter to obtain more discharge.

* A triangular channel section will be most economical or most efficient when each of its slopping sides makes an angle of 45° with the vertical.

* The hydraulic radius R of a triangular channel section can be expressed as,

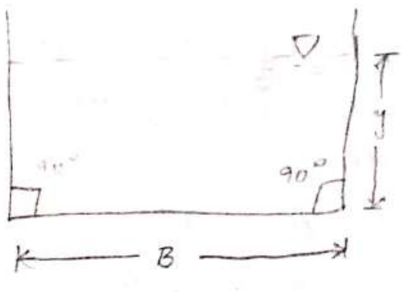
$$R = \frac{A}{P} = \frac{y^2\tan\theta}{2y\sec\theta}$$

Sub θ in the above equation, we get

$$R = \frac{y}{2\sqrt{2}} \rightarrow ⑤$$

* Thus it can be seen that the most economical or most efficient triangular channel section will be half square described on a diagonal & having equal slopping sides.

Most economical/Most efficient Rectangular Channel section:



Area, $A = B \times y \Rightarrow B = \frac{A}{y}$

Perimeter, $P = 2y + B \Rightarrow P = 2y + \frac{A}{y}$

To get most economical or most efficient channel section, we have to satisfy this condition.

$$\frac{\partial P}{\partial y} = 0$$

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$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\left[\frac{A}{y} + 2y\right]$

$= A\left[\frac{-1}{y^2}\right] + 2$

$= -\frac{A}{y^2} + 2 \Rightarrow \frac{A}{y^2} = 2$

$A = 2y^2$

$B \times y = 2y^2$

$B = 2y$

$y = \frac{B}{2}$

Area $A = 2y^2$
Breadth $B = 2y$
Perimeter $P = B + 2y$
Hy. Radius $R = \frac{y}{2}$
Hy. Depth $D = y$

Conditions for a Rectangular channel

$P = B + 2y = 2y + 2y = 4y$

Hydraulic Radius,
$R = \frac{A}{P} = \frac{2y^2}{4y} = \frac{y}{2}$

Hydraulic depth, $D = \frac{A}{T}$

$= \frac{2y^2}{B}$

$= \frac{2y^2}{2y}$

$\boxed{D = y}$

Q. Resistance Coefficient $n = 0.018$

Discharge $q = 5.07\ m^3/sec/m$

Slope $S = \frac{1}{1000}$

$\because R = \frac{y}{2}$; $B = 2y$

By using Manning's equation,

$Q = \frac{q}{B}$

$Q = B \times y \times \frac{1}{n} R^{2/3} S^{1/2}$

$Q = y \times \frac{1}{n} (y/2)^{2/3} S^{1/2}$

$5.07 = \frac{1}{0.018} \times \frac{1}{(2)^{2/3}} \times y^{5/3} \times \left[\frac{1}{1000}\right]^{1/2}$

$5.07 = 1.1067\ y^{5/3}$

$y = 2.49 \approx 2.5m$

$\therefore y = 2.5\ m$

$B = 2y = 2(2.5) = 5m$

$P = B + 2y = 5 + 2(2.5) = 10\ m.$

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Trapezoidal Channel :

By Pythagoras theorem
$(hyp)^2 = (\perp)^2 + (Base)^2$

$AB^2 = y^2 + (zy)^2$

$AB = \sqrt{y^2 + z^2 y^2}$

$AB = y\sqrt{1+z^2}$ $A = \frac{1}{2}(a+b)h$

$Area = \frac{1}{2}\left[B + 2zy + B\right] \times y$

$= \frac{1}{2}\left[2B + 2zy\right]y$

$= \frac{1}{2} \times 2\left[B + zy\right]y$

$$\boxed{\begin{aligned} A &= (B+zy)y \\ B &= \frac{A}{y} - zy \end{aligned}}$$

Wetted Perimeter, $\boxed{P = 2y\sqrt{z^2+1} + B}$

$$\boxed{P = 2y\sqrt{z^2+1} + \frac{A}{y} - zy}$$

$\frac{\partial P}{\partial y} = 0$

$\frac{\partial}{\partial y}\left[2y\sqrt{z^2+1} + \frac{A}{y} - zy\right] = 0$

$\frac{\partial}{\partial y}\left(2y\sqrt{z^2+1}\right) + \frac{\partial}{\partial y}\left(\frac{A}{y}\right) - \frac{\partial}{\partial y}(zy) = 0$

$2(1)\sqrt{z^2+1} - \frac{A}{y^2} - z(1) = 0$

$2\sqrt{z^2+1} = \frac{A}{y^2} + z$

$2\sqrt{z^2+1} = \frac{(B+zy)y}{y^2} + z$

$\frac{d}{dy}(y^{-1}) = -1y^{-1-1} = -y^{-2} = -\frac{1}{y^2}$

$2\sqrt{1+z^2} = \frac{B+zy}{y} + \frac{z}{1}$

$2\sqrt{1+z^2} = \frac{B+zy+zy}{y} \Rightarrow 2y\sqrt{1+z^2} = B+2zy$

$$\boxed{y\sqrt{1+z^2} = \frac{B+2zy}{2}} \rightarrow \text{condition } 1$$

Slope $i = \frac{T}{2}$

Hydraulic Mean Depth $m = \frac{A}{P}$

$m = \frac{(B+zy)y}{2y\sqrt{z^2+1} + B}$

$= \frac{(B+zy)y}{(B+2zy)+B} = \frac{(B+zy)y}{B+2zy+B} = \frac{(B+zy)y}{2B+2zy} = \frac{(B+\cancel{zy})y}{2(B+\cancel{zy})}$

$$\boxed{m = \frac{y}{2}} \rightarrow \text{condition } 2$$

θ = Angle made by the sloping side with horizontal

O = The centre of the top width, AD

Drawn $\perp$ to sloping side AB

$\sin\theta = \frac{OF}{OA}$

In $\triangle AEB$, $\sin\theta = \frac{AE}{AB} = \frac{y}{y\sqrt{z^2+1}}$

$\sin\theta = \frac{1}{\sqrt{z^2+1}}$

$\frac{1}{\sqrt{z^2+1}} = \frac{OF}{OA}$

$OF = \frac{OA}{\sqrt{z^2+1}}$

$OA = \frac{B+2zy}{2} = y\sqrt{z^2+1}$

Sub OA in the above eqn.

$OF = \frac{y\sqrt{z^2+1}}{\sqrt{z^2+1}} = y$

$\boxed{OF = y}$ where y = depth of flow

The 3 sides of most economical trapezoidal section will be tangential to the semi circle.

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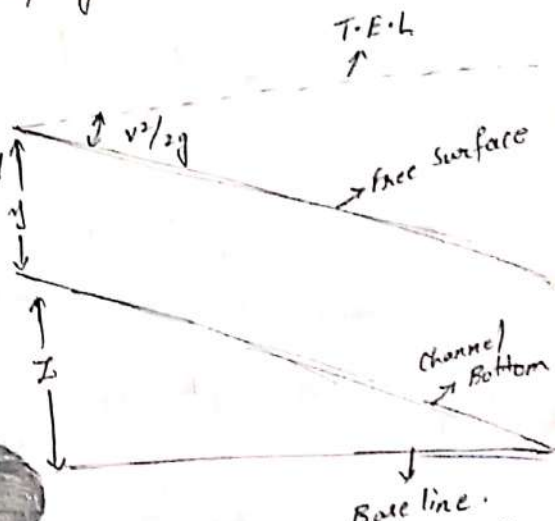
3. Non Uniform Flow in Open Channels

Gradually Varied Flow : In this case of flow, the depth of flow increases or decreases gradually in the direction of flow. This change from one depth of flow to another occurs gradually in a distance of applicable length.

Rapidly Varied Flow : In this case a sudden change of depth occurs at a particular point of a channel within a very short length is known as rapidly varied flow.

Specific Energy : It is defined as energy measured w.r.to channel bed as datum.



$$\text{Total Energy} = \frac{z + y + \frac{v^2}{2g}}{\text{unit weight of flow}}$$

y : Depth of flow

v : Avg. velocity of flow

z : Datum or Elevation of the channel above the horizontal surface.

* The energy per unit weight of flowing fluid above the channel bottom although the total energy is reduced by friction, the specific energy can increase or decrease from section to section if bed elevation changes, however for uniform flow specific energy is constant. If the channel bottom is itself taken as a datum line then the total energy per unit width is $E = y + \frac{v^2}{2g} \Rightarrow \boxed{E = E_p + E_K}$

Let us consider a rectangular section of b = Width of channel

y = Depth of flow

Q = Discharge through the channel.

Now velocity of flow, $V = \frac{Q}{A} = \frac{q \times b}{b \times y} = \frac{q}{y}$

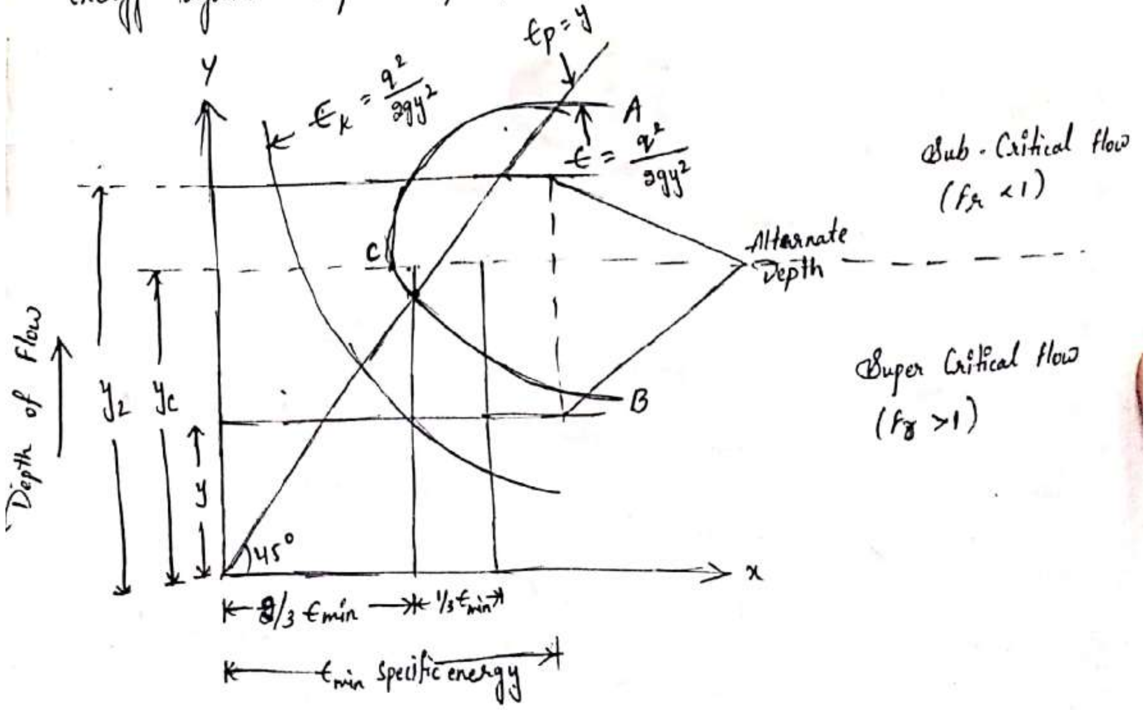
$$E = y + \frac{V^2}{2g}$$

$$= \frac{y + \left(\frac{q}{y}\right)^2}{2g}$$

$$= y + \frac{\frac{q^2}{y^2}}{2g}$$

$$\boxed{E = y + \frac{q^2}{2gy^2}} \rightarrow \text{gate}$$

In the above equation, if it is represented graphically then it is known as specific energy curve. It consists of specific energy against depth of flow.



* It plots a curve of potential energy with a straight passing curve through the origin making an angle of 45° with x & y axes.

The ... is a parabola plot for specific energy obtained by adding kinetic & potential energy.

It shows that the type of flow i.e., sub-critical flow, critical flow & super critical flow.

It shows the corresponding depths.

It clearly shows that occurence of min. specific energy where depth is critical depth.

Critical Depth (y_c) :- In the above curve ACB is the critical depth y_c. The depth of flow at which where specific energy is minimum is called critical depth.

Critical Depth is obtained by differentiating specific energy equation wrt y & it is equal to zero.

$$\frac{dE}{dy} = 0$$

$$\frac{d}{dy}\left[y + \frac{q^2}{2gy^2}\right] = 0$$

$$\frac{d}{dy}(y) + \frac{d}{dy}\left[\frac{q^2}{2gy^2}\right] = 0$$

$$1 + \frac{q^2}{2g}\frac{d}{dy}\left[y^{-2}\right] = 0$$

$$1 + \frac{q^2}{2g}\left[-\frac{2}{y^3}\right] = 0$$

$$1 - \frac{\cancel{2}q^2}{\cancel{2}gy^3} = 0$$

$$\frac{q^2}{gy^3} = 1$$

$$q^2 = gy^3$$

$$y^3 = \frac{q^2}{g}$$

$$y = \left[\frac{q^2}{g}\right]^{1/3}$$

But where sp. energy is min, it is called Cr. depth.

So $y = y_c$

$$\boxed{y_c = \left[\frac{q^2}{g}\right]^{1/3}} \longrightarrow \text{Gate}$$

<u>Critical velocity</u> : The velocity of flow at critical depth is known as critical velocity.

$$Q = AV$$

$$V = \frac{Q}{A}$$

$$V_c = \frac{q}{y_c} = \frac{q}{\left(\frac{q^2}{g}\right)^{1/3}} = q^1 \times q^{-2/3} \times g^{+1/3}$$

$$V_c = \cancel{\frac{q}{g}} \quad q^{1/3} \cdot g^{1/3}$$

$$V_c^3 = q \cdot g \qquad \because q = V_c \times y_c$$

$$\cancel{V_c^3 = \left[\frac{q}{g}\right]^{1/3}}$$

$$\cancel{V_c^3 = \frac{q}{g}}$$

$$\cancel{V_c^3 = \frac{V_c \times y_c}{g}}$$

$$\cancel{V_c^2 = \frac{y_c}{g}}$$

$$V_c = \sqrt{\frac{y_c}{g}}$$

$$V_c^{\cancel{3}} = \cancel{V_c} \times y_c \times g$$

$$\cancel{\because q = V_c \times y_c}$$

$$V_c^2 = y_c \times g$$

$$V_c = \sqrt{y_c \times g}$$

$$\boxed{\frac{V_c}{\sqrt{y_c \times g}} = 1}$$

Minimum specific energy in terms of critical depth.

$$E = y + \frac{q^2}{2gy^2}$$

$$E_{min} = y_c + \frac{q^2}{2g y_c^2}$$

$$= y_c + \frac{V_c^2 \times \cancel{y_c^2}}{2g\,\cancel{y_c^2}}$$

$$= y_c + \frac{V_c^2}{2g}$$

$$= y_c + \frac{y_c \times \cancel{g}}{2\cancel{g}}, \quad \frac{2y_c + y_c}{2} = \frac{3y_c}{2}$$

$$E_{min} = \frac{3}{2} y_c$$

$$y_c = \frac{2}{3} E_{min}$$

Critical Flow: A critical flow is one in which sp. energy is min, A flow corresponding to critical depth is also known as critical flow.

i.e., $\frac{V_c}{\sqrt{g \times y_c}} = 1$

Subcritical Flow: The flow is subcritical or streaming flow or ~~tring~~ tranquill flow when depth of flow in a channel is greater than y_c. Also called as shooting flow.

i.e., $y > y_c$ or Fr No < 1 then $V < V_c$

Super Critical Flow: The flow is super critical or shooting or torrential flow when the depth of flow of channel is less than critical depth.

i.e., $y < y_c$ or Fr No > 1 then $V > V_c$

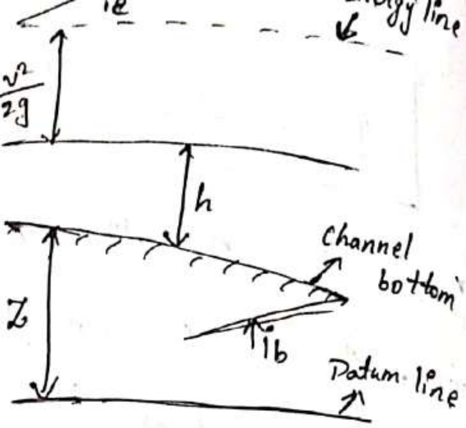
Dynamic Equation for Gradually Varied Flows:

If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow.

Assumptions:

1. The bedslope of the channel is small.
2. The flow is steady & hence discharge is constant.
3. The compaction factor is unity i.e., Kinetic correction factor.
4. The formulas such as chezy's & manning's which are applicable to uniform flow is also applicable.
5. The roughness coefficient is constant.
6. The channel is prismatic.

Consider a rectangular channel having gradually varied flow as shown in fig, the depth of flow is gradually decrease in the direction of flow



Let z = Height of channel bottom above the datum line

h = Depth of flow.

v = Mean Velocity of flow

q = Discharge per unit width.

i_b = Slope of channel bed

i_e = Slope of energy line

b = Width of channel

Q = Discharge through the channel.

The energy equation at any section is given by bernoullis equation

i.e., $E = z + h + \frac{v^2}{2g} \rightarrow (1)$

Differentiate the above equation wrt x.

$$\frac{dE}{dx} = \frac{d}{dx}\left(z + h + \frac{v^2}{2g}\right)$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

from $Q = AV$

$$V = \frac{Q}{A}$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{Q^2}{A^2 \times 2g}\right) \qquad \because A = b \times h$$

$$= \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left[\frac{Q^2}{b^2 \times h^2 \times 2g}\right]$$

$$= \frac{dz}{dx} + \frac{dh}{dx} + \frac{Q^2}{b^2 \times 2g}\frac{d}{dx}\left[\frac{1}{h^2}\right]$$

$$= \frac{dz}{dx} + \frac{dh}{dx} + \frac{Q^2}{b^2 \times \cancel{2}g} \cdot \frac{-\cancel{2}}{h^3}\frac{dh}{dx}$$

$$= \frac{dz}{dx} + \frac{dh}{dx} - \frac{Q^2}{b^2 h^3 g} \cdot \frac{dh}{dx}$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx} - \frac{q^2}{b^2 \times h^2 \times gh} \times \frac{dh}{dx}$$

$$= \frac{dZ}{dx} + \frac{dh}{dx} - \frac{q^2}{A^2 \times gh} \times \frac{dh}{dx}$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx}\left[1 - \frac{q^2}{A^2 \times gh}\right]$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right] \rightarrow (2)$$

Now $\frac{dE}{dx} = -i_e$

$\frac{dz}{dh} = -i_b$.

-ve of i_e & i_b is taken as with the increase of value of x & decrease in values of E & z. sub $\frac{dE}{dx}$ & $\frac{dZ}{dx}$ in eq (2)

$$-i_e = -i_b + \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right]$$

$$i_b - i_e = \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right]$$

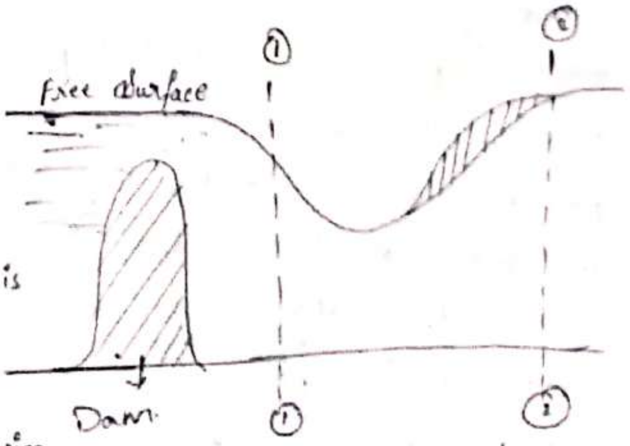
$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{V^2}{2g}}$$

$$F_r = \sqrt{\frac{V}{gh}}$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - F_r^2}$$

where $\frac{dh}{dx}$ = variation of depth of flow along length.

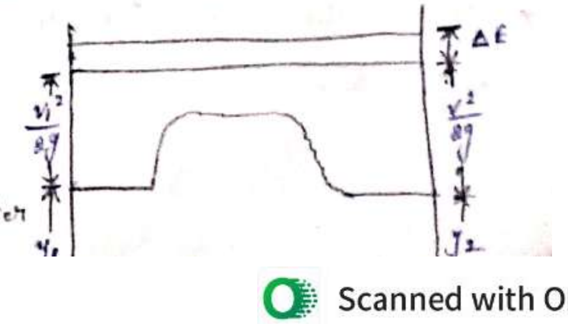
Hydraulic Jump:



Consider the flow of water at section 1-1 over a dam as shown in fig.

The height of water @ section 1 is small, as we move towards downstream, the height or the depth of water is rapidly increasing over a short length of channel.

This is because @ section 1 - the flow is super critical as the depth of water



at section 1-1 less than critical depth.

* Super Critical Flow is unstable flow & does not continue on the downstream side.

* When these super critical flow will convert itself into a streaming flow, hence, the depth of water will increase.

* This sudden increase of depth or height of water is called a hydraulic jump.

Assumptions for analysing of hydraulic jump:

The following are the assumptions made for analysing the hydraulic jump:

* It is assumed that before & after the jump, the flow is said to be uniform flow.

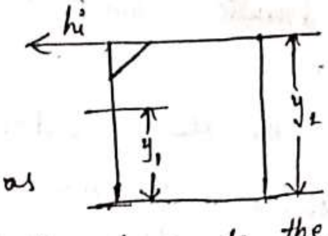
* The length or jump is small so that the losses due to friction on the channel flow are small & hence neglected.

* The channel flow is horizontal. The height components of water comprising the jump is negligably small.

Properties of Hydraulic Jump:

2M.

1. **Length of Jump (L_j):**



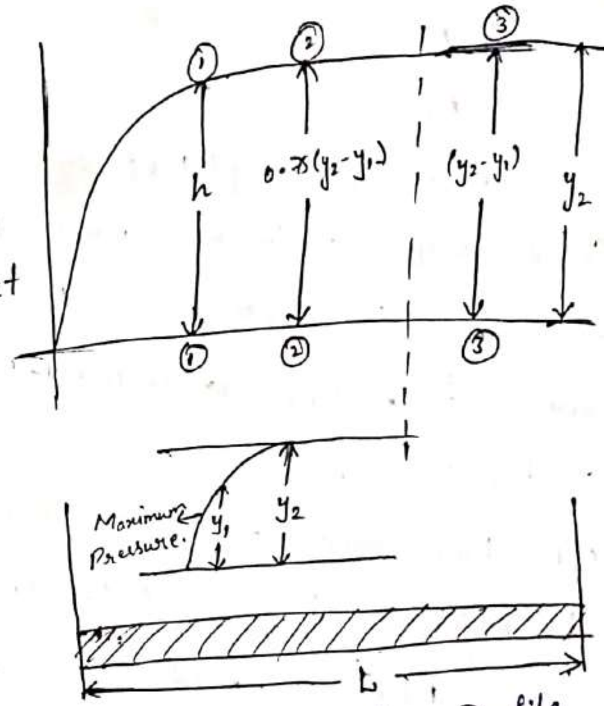
Length of jump is an important parameter effecting the sight of basin in which the jump is used to take the length of jump as the horizontal distance b/w the toe of the jump to the section where water surface reaching the depth can be expressed as: $\boxed{L_j = 6.9(y_2 - y_1)}$

* It is the difference b/w max to the point where hydraulic jump ends.

2. Pressure Distribution : The pressure distribution of the toe of the jump & at the end of the jump having same pressure distribution. At the inside of the body of the toe jump the strong curvature of stream lines ~~of pressure to~~ deviate from hydrostatic pressure & increase in the froude number.

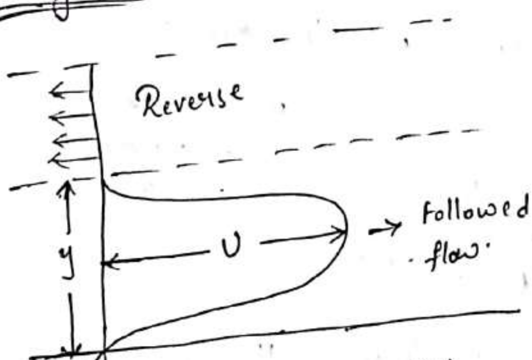
3. Water Surface Profile :

* By bed slope we can design the slope of the channel.
* By this hydraulic jump at certain length we can construct the walls of the channel.



Water Surface Profile

Velocity Surface Profile :

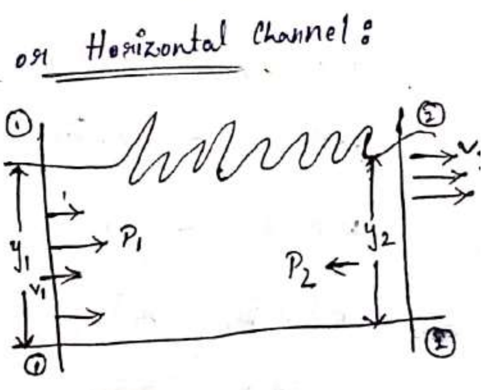


Velocity Surface Profile.

* from toe to high velocity in hydraulic jump, the flow is rotational & it can be irreversal from sub-critical state to high velocity state.

Hydraulic Jump in Rectangular channel or Horizontal Channel :

It is defined as the rapid variation of flow from a super critical state, the depth of flow before the jump is known as initial depth & it is denoted by 'y_1'. The depth of flow after the jump is known as final depth & it is denoted by 'y_2'.



... channel of horizontal bed.

* Consider two sections 1-1 & 2-2 before & after the jump.

Let y_1 : Depth of flow @ 1-1

y_2 : Depth of flow @ 2-2

v_1 : velocity @ 1-1

v_2 : velocity @ 2-2

A_1 = c/s Area @ 1-1

A_2 = c/s Area @ 2-2

$\bar{z}_1$ & $\bar{z}_2$ = Depths of centroid or centroid depths at 1-1 & 2

Consider unique depth of channel, the forces acting on mass of water b/w two sections i.e., 1-1 & 2-2.

Pressure force (p_1) @ section 1-1 & Pressure force (p_2) @ section 2-2.

* Frictional area on the floor of channel which assumed to be neglected. Hence in accordance with the momentum equation

$$f(dt) = m(dv) \qquad m = \rho \times V = \rho \times A \times L = \rho Q$$

$$P_2 - P_1 \, (dt) = \rho Q \, dv$$

$$P_2 - P_1 \, (dt) = \rho Q \, (v_1 - v_2) \rightarrow (1)$$

- Incase of hydrostatic pressure distribution, the pressure force at section 1-1 i.e., $P = \omega A \bar{z} \rightarrow (2)$

Sub eq (2) in eq (1)

$$\omega A_2 \bar{z}_2 - \omega A_1 \bar{z}_1 \, (dt) = \rho Q (v_1 - v_2)$$

$$\omega A_2 \bar{z}_2 - \omega A_1 \bar{z}_1 = \rho Q \left[\frac{Q}{A_1} - \frac{Q}{A_2} \right]$$

$$\omega A_2 \bar{z}_2 - \omega A_1 \bar{z}_1 = \frac{\omega}{g} \cdot Q \left[\frac{Q}{A_1} - \frac{Q}{A_2} \right]$$

$$\omega A_2 \bar{z}_2 - \omega A_1 \bar{z}_1 = \frac{\omega}{g} \left[\frac{Q^2}{A_1} - \frac{Q^2}{A_2} \right]$$

$$\cancel{\omega} \left[A_2 \bar{z}_2 - A_1 \bar{z}_1 \right] = \frac{\cancel{\omega}}{g} \left[\frac{Q^2}{A_1} - \frac{Q^2}{A_2} \right]$$

$$A_2 \bar{z}_2 - A_1 \bar{z}_1 = \frac{1}{g} \left[\frac{Q^2}{A_1} - \frac{Q^2}{A_2} \right]$$

$$A_2 \bar{z}_2 - A_1 \bar{z}_1 = \frac{Q^2}{A_1 g} - \frac{Q^2}{A_2 g}$$

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$$\boxed{A_2\bar{z}_2 + \frac{Q^2}{A_2 g} = A_1\bar{z}_1 + \frac{Q^2}{A_1 g}} \rightarrow (3)$$

$A = B \times D$

$A_{(1)} = B \times y_1 = 1 \times y_1 = y_1$

$A_{(2)} = B \times y_2 = 1 \times y_2 = y_2$

$\bar{z}_1 = \frac{y_1}{2}$; $\bar{z}_2 = \frac{y_2}{2}$

Sub in eq (3)

$$\frac{Q^2}{gA_2} + y_2 \times \frac{y_2}{2} = \frac{Q^2}{gA_1} + y_1 \times \frac{y_1}{2}$$

$$\frac{Q^2}{g y_2} + \frac{y_2^2}{2} = \frac{Q^2}{g y_1} + \frac{y_1^2}{2}$$

$$\frac{Q^2}{g y_2} - \frac{Q^2}{g y_1} = \frac{y_1^2}{2} - \frac{y_2^2}{2}$$

$$\frac{Q^2}{g}\left[\frac{1}{y_2} - \frac{1}{y_1}\right] = \frac{1}{2}\left[y_1^2 - y_2^2\right] \rightarrow (4)$$

For unit width, $q = \frac{Q}{B}$

$q = \frac{Q}{1}$

$\boxed{q = Q}$

$$\frac{q^2}{g}\left[\frac{1}{y_2} - \frac{1}{y_1}\right] = \frac{1}{2}\left[y_1^2 - y_2^2\right]$$

$$\frac{q^2}{g}\left[\frac{y_1 - y_2}{y_1 y_2}\right] = \frac{1}{2}\left[y_1^2 - y_2^2\right] \rightarrow (5)$$

$$\frac{2q^2}{g}\left[\frac{\cancel{y_1 - y_2}}{y_1 y_2}\right] = (y_1 + y_2)\cancel{(y_1 - y_2)}$$

$$\frac{2q^2}{g(y_1 y_2)} = y_1 + y_2$$

$$\frac{2q^2}{g} = (y_1 + y_2)(y_1 y_2)$$

Relation b/w Q & q & sequent depths in eq (5) is the momentum equation for rectangular channel.

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$= y_1^2 y_2 + y_1 y_2^2$$

$$\frac{2q^2}{g} = y_1(y_1 y_2 + y_2^2)$$

$$\frac{2q^2}{g y_1} = y_1 y_2 + y_2^2$$

$$y_1 y_2 + y_2^2 - \frac{2q^2}{g y_1} = 0 \rightarrow (6)$$

$$y_2^2 + y_1 y_2 - \frac{2q^2}{g y_1} = 0 \qquad ax^2 + bx + c = 0$$

Eq (6) is a quadratic eq in terms of y_2.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-y_1 \pm \sqrt{y_1^2 + 4(1) \times \frac{2q^2}{g y_1}}}{2(1)}$$

$$= \frac{-y_1 \pm \sqrt{y_1^2 + \frac{8q^2}{g y_1}}}{2}$$

Hence the two roots are

$$y_2 = \frac{-y_1}{2} - \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

$$y_2 = \frac{-y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

Negative Root is neglible. Consider only +ve root.

$$\frac{q^2}{g} = y_c^3$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\left(y_c/y_1\right)^3}\right] \rightarrow (7)$$

$$\boxed{y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8(F_r)^2} - 1\right]}$$

Expression for loss of energy due to hydraulic jump:

The energy loss in hydraulic jump due to formation of eddies & turbulent occurs this loss of energy is computed at 2 sections as $h_j = E_1 - E_2$ or $\Delta E = E_1 - E_2$

From continuity eq, $Q = A_1 V_1 = A_2 V_2$

$$Q = B \times y_1 \times V_1 = B \times y_2 \times V_2$$

$$V_1 = \frac{Q}{B \times y_1} \quad ; \quad V_2 = \frac{Q}{B \times y_2}$$

$$\Delta E = E_1 - E_2$$

$$= \left[y_1 + \frac{Q^2}{2g(y_1)^2} \right] - \left[y_2 + \frac{Q^2}{2g(y_2)^2} \right]$$

$$\Delta E = \left[\frac{q^2}{2gy_1^2} - \frac{Q^2}{2gy_2^2} \right] - (y_2 - y_1)$$

$$= \left[\frac{q^2}{2gy_1^2} - \frac{q^2}{2gy_2^2} \right] - (y_2 - y_1)$$

$$\Delta E = \frac{q^2}{2g} \left[\frac{1}{y_1^2} - \frac{1}{y_2^2} \right] - (y_2 - y_1)$$

On simplication, we get

$$\boxed{\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}}$$

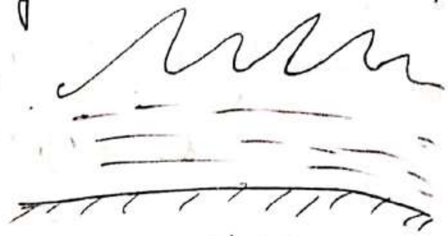
Energy loss in terms of velocity → Characteristic of Hydraulic jump.

$$\boxed{\Delta E = \frac{(V_1 - V_2)^2}{2g(V_1 + V_2)}}$$

where V_1 & V_2 are velocities of flow before & after the jump.

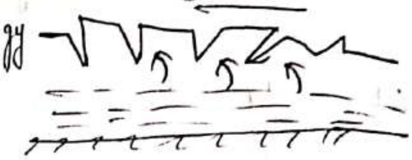
Types of Hydraulic Jump : Occurs on hydraulic jump. They are:

a. Undular Jump : For froude no. 1.0 - 1.7 , the water surface undergo some undulations & the jump is called undular jump. These waves are small and gradually diminished in amplitude.



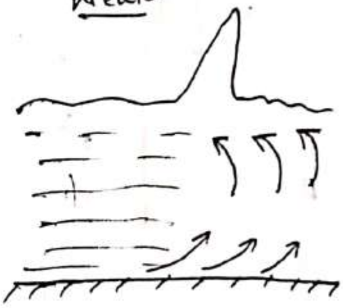
Undular.

b. Weak Jump : For froude no 1.7 - 2.5, the jump formed is called weak jump as the velocity through out the section is uniform & only a small amount of energy is dissipated.



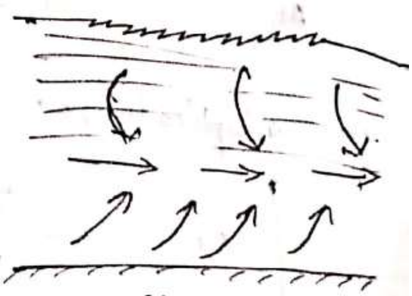
Weak

c. Oscillating Jump : For Fr no 2.5 - 4.5 , jump is formed called oscillating jump. In this case, the entering of jet of water oscillating back - forth & bottom - to the surface & back against the formation of large waves of irregular period.



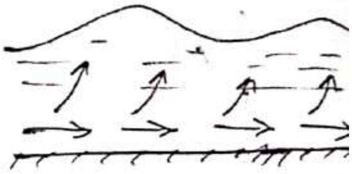
Oscillating

d. Steady Jump : For Fr No 4.5 - 9 , the jump formed is well stabilized & it is called a steady jump. For this jump, the energy dissipation rises from 45 - 75%.



Steady

e. Strong Jump : For Fr.No > 9 , the large jump formed is known as strong jump. & the energy dissipation ranges upto 85%.



Strong

Applications of Hydraulic Jump :

- It raises water level in the channel for irrigation purpose etc..,
- It increases the discharge through a sluice by holding back the tail end.
- It maybe used for mixing chemicals in water & other liquids.
- To operate flow measurement efficiently.
- It acts as energy dissipation to dissipate the excess energy of water flowing from downstream of spillways & sluicegates etc..,
- In desalination of sea water.

Characteristics of Hydraulic Jump :

Conjugate Depth : The Conjugate depth of the hydraulic jump are y_1 & y_2, because these are the depths above & below the critical depth.

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(Fr)^2} - 1\right]$$

$$Fr = \sqrt{\frac{v}{gy_1}}$$

y_1 = Depth of supercritical flow

y_2 = Depth of sub critical flow

v_1 = velocity of super critical flow

g = Acceleration due to gravity

Length of Jump (L_j) : It may defined as the distance measured from the front face of the jump to a point on the surface immediately downstream from the roller.

Height of Jump (h_j) : It is equal to difference b/w the depths before & after the jump..

$$h_j = h_2 - h_1 \text{ or } y_2 - y_1$$

Location of hydraulic jump: A hydraulic jump is formed whenever the momentum equation is satisfied b/w super critical & sub-critical flow of a stream in connection with uniformly varied flow. calculations. It has already indicated that the control for super critical flow control is at upstream end & sub critical flow control is at downstream end. The hydraulic jump must satisfy all these requirements.

① The depth of flow of water at a certain section of a rectangular channel at 4m wide is 0.5m. The discharge through the channel is $16\,m^3/sec$. If a hydraulic jump takes place on the downstream side. Find the depth of flow after the jump.

Sol: Given that:

Depth of channel $d_1 = 0.5m$

Width of channel $b = 4m$

Discharge of channel $Q = 16m^3/sec$

Discharge/width $q = \frac{Q}{b} = \frac{16}{4} = 4\,m^3/sec$

Let depth of flow after the jump is d_2

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd}}$$

$$d_2 = -\frac{0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2(4)^2}{9.81 \times 0.5}}$$

$$= 2.316\ m$$

② The depth of flow of water at a rectangular section of 2m wide is 0.3m ; The discharge through the channel is $1.5\,m^3/sec$. Determine whether the hydraulic jump will occur & if so find its height & loss of energy per Kg of water.

Sol: Width of channel $b = 2m$

Depth of channel $d_1 = 0.3m$

Discharge of channel $Q = 1.5m^3/se$

Discharge/width $q = \frac{Q}{b} = \frac{1.5}{2} = 0.75\,m^3/sec.$

Hydraulic jump will occur if the depth of flow on the upstream side is less than the critical depth on upstream side or if the Fr. No on upstream side is more than 1.

Critical depth $h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.75^2}{9.81}\right)^{1/3} = 0.48m$

The depth on upstream side is 0.3m, hence it is less than the critical depth on upstream side., therefore hydraulic jump will occur.

Depth of flow after hydraulic jump,

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2(0.75)^2}{9.81 \times 0.3}}$$

$$d_2 = 0.486 \, m$$

Height of hydraulic jump, $h_j = d_2 - d_1 = 0.486 - 0.3 = 0.186$

Energy loss due to hydraulic jump,

$$\Delta E = \frac{(d_2 - d_1)^3}{4 d_1 d_2} = \frac{(0.486 - 0.3)^3}{4 \times 0.3 \times 0.486} = 0.011 \text{ m-Kg/Kg}.$$

) A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10m/sec & depth of flow is 1m. Determine the depth after the jump & consequent loss in total head.

Sol: Given that:

Velocity of flow before & after the jump $v_1 = 10m/sec$

Depth of flow before jump $d_1 = 1m$

Discharge/width, $q = v_1 \times d_1 = 10 \times 1 = 10 m^3/sec$.

Depth of flow after jump $d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$

$$= -\frac{1}{2} + \sqrt{\frac{1^2}{4} + \frac{2(10)^2}{9.81 \times 1}}$$

$$= 4.04 m$$

Loss in total head, $h_L = \frac{(d_2 - d_1)^3}{4 d_1 d_2} = \frac{(4.04 - 1)^3}{4 \times 1 \times 4.04} = 1.73m.$

(4) A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6m/sec & depth of flow is 0.4m. The width of the channel is 8m. Determine whether a hydraulic jump will occur & if so find its height & loss of energy per kg of water. Also determine power loss in the hydraulic jump.

$Fr_1 = \frac{v}{\sqrt{gy}} = \frac{6}{\sqrt{9.81 \times 0.4}} = 3.03$ Super critical flow

$(y = d_1)$

$Fr_1 > 1$, Hence hydraulic jump will occur

Sol: Velocity of flow $v_1 = 6 m/sec$

Depth of channel flow $d_1 = 0.4m$

Width of channel $b = 8 m$

$q = v_1 \times d_1 = 6 \times 0.4 = 2.4 \; m^3/sec$

Critical Depth $h_c = \left[\frac{q^2}{g}\right]^{1/3} = \left[\frac{2.4^2}{9.81}\right]^{1/3} = 0.837 \; m$

The depth on upstream side is 0.4m, hence it is less than the critical depth on upstream side, ∴ hydraulic jump will occur.

Depth of flow after hydraulic jump,

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.4}{2} + \sqrt{\frac{0.4^2}{4} + \frac{2(2.4)^2}{9.81 \times 0.4}} = 1.52 \; m$$

Height of hydraulic jump, $h_j = d_2 - d_1 = 1.52 - 0.4 = 1.12 \; m$

Energy loss due to hydraulic jump,

$$\Delta E = \frac{(d_2 - d_1)^3}{4 d_1 d_2} = \frac{(1.52 - 0.4)^3}{4 \times 1.52 \times 0.4} = 0.577 \; m.Kg/Kg.$$

Power Loss in the hydraulic jump:

P_{loss} = Weight flow rate × Energy loss

$= \gamma Q \times \Delta E$

$= \gamma (b y_1 V) \times \Delta E$

$= 9810 (8 \times 0.4 \times 6) \times 0.577$

$= 108.68 \; KW$

4th-Unit

4th Unit

Impact of jet on vanes:- The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure.

* If some plate which may be fixed or moving is placed in the path of the jet, a force exerted by the jet on

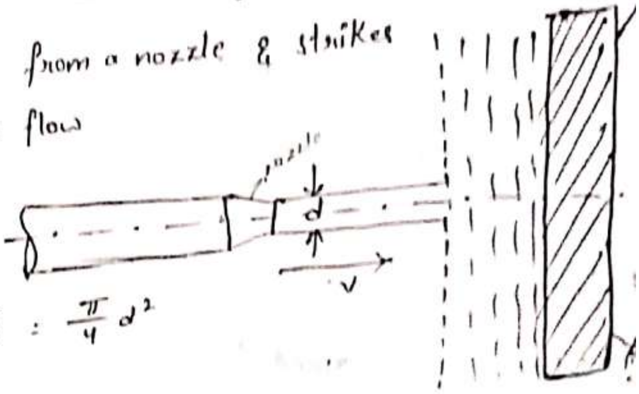
...anes, this ... on impulse momentum equation. This impact of ... the force exerted by a jet on a plate which is stationary or moving.

Types of Vanes :

- Stationary / fixed
  - Vertical / flat
  - Inclined
  - Curved
- Moving
  - Vertical / flat
  - Inclined
  - Curved.

Force exerted by a Jet on a stationary vertical plate :

Consider a jet of water coming out from a nozzle & strikes a fixed vertical plate ⊥ to the flow



Let v = velocity of jet

d = Diameter of jet

a = Area of cross section of jet = $\frac{\pi}{4} d^2$

w = sp. wt of water = ρg

The jet after striking the plate will move along the plate & gets deflected to 90°.

Velocity after striking the plate is zero '0'.

According to impulse momentum principle,

force exerted in the direction of flow on the vane is

f_x = Rate of change of momentum in the direction of flow.

Then, $F_x = \frac{\text{Initial Momentum} - \text{Final Momentum}}{\text{Time}}$

$= \frac{\text{Mass} \times \text{Initial Velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$

$= \frac{\text{Mass}}{\text{Time}} [\text{Initial Velocity} - \text{Final velocity}]$

$= \frac{\text{Mass}}{\text{time}} [v - 0]$

$= \frac{\rho \times \text{volume}}{\text{time}} [v - 0]$

$\frac{\quad}{time}[v]$

$= \rho \times A \times v\,(v)$

$$\boxed{F_x = \rho A V^2}$$

$$F_y = \frac{Mass}{Time}\left[\text{Initial velocity} - \text{Final velocity}\right]$$

$$= \frac{Mass}{Time}[0-0]$$

$$\boxed{F_y = 0}$$

Work done = force × distance

Work done/sec = Force × distance/sec

$= \rho A V^2 \times 0$

$$\boxed{\text{Work done per sec} = 0}$$

Efficiency is the ratio of useful work performance by a machine to a total energy expended.

$$\text{Efficiency } \eta = \frac{\text{Work done}}{K.E} = \frac{0}{\frac{1}{2}mv^2} = 0$$

Force exerted by a jet on moving vertical plate :

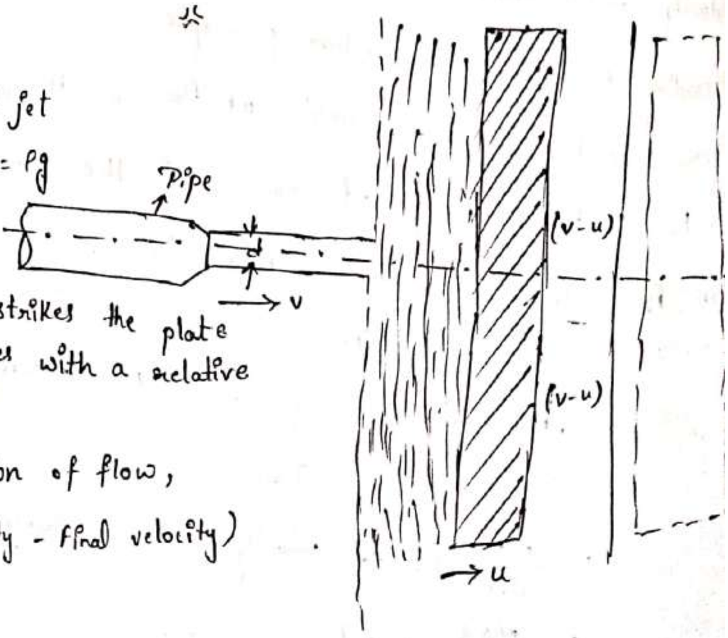
Let V = Velocity of jet

d = Dia of jet

A = Area of c/s of jet

w = Sp. wt of water = ρg

u = Velocity of plate



In this case, jet does not strikes the plate with velocity v, but it strikes with a relative velocity $(v-u)$.

Force acting in the direction of flow,

$$F_x = \frac{Mass}{time}(\text{Initial velocity} - \text{Final velocity})$$

$$= \rho A V\,((v-u) - 0)$$

$$= \rho A (v-u)\,(v-u)$$

$$\boxed{F_x = \rho A(v-u)^2}$$

$F_y = 0$

Work done = force x Distance

$$\boxed{\text{Work done} = \rho A(v-u)^2 \times u}$$

Efficiency $\eta = \dfrac{\text{Work done per sec}}{K.E}$

$$\boxed{\eta = \dfrac{\rho A(v-u)^2 \times u}{\frac{1}{2} m v^2}}$$

1) Water is flowing through a pipe at the end of which a nozzle is fitted, the diameter of nozzle is 100mm & head of water at the central nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. Coefficient of velocity is given as $C_v = 0.95$.

Sol: Given Data : Dia of nozzle, d = 100mm = 0.1 m

Head of water, H = 100m

Coefficient of velocity, C_v = 0.95

Force exerted by the jet of water = $\rho A V^2$

$F_x = 1000 \times \frac{\pi}{4}(0.1)^2 \times (42.08)^2$

$F_x = 13900.6\,N$

$C_v = \dfrac{\text{actual velocity}}{\text{theoritical velocity}}$

$0.95 = \dfrac{\text{act velocity}}{\sqrt{2gh}}$

$v = 0.95 \times \sqrt{2 \times 9.81 \times 100} = 42.08\,m/sec$

2) A jet of water 120mm dia moving with a velocity of 25m/sec strikes the flat vertical plate normally, determine the force exerted on the flat plate & workdone for : i) When the plate is fixed ii) When the plate is moving with a velocity of 8m/sec in the direction of jet. iii) Plate is moving with a velocity of 8m/sec towards the jet.

Sol: Given Data : Dia of water, d = 120mm = 0.12 m

Velocity of jet, v = 25 m/sec

Velocity of plate, u = 8m/sec

Area of jet, a = $\frac{\pi}{4}(0.12)^2$ = 0.0113 m^2

i) $F_x = \rho A V^2 = 1000 \times 0.0113 \times 25^2 = 7062.5\,N$

Work done = 0

ii) $F_x = \rho A(v-u)^2 = 1000 \ldots$

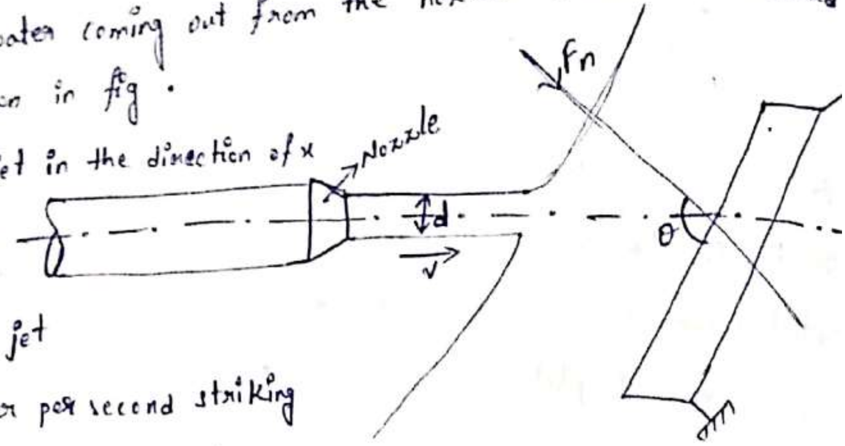
Work done $= \rho A(v-u)^2 \times u = 3265.7 \times 8 = 26125.6$ N

iii) $F_x = \rho A(v+u)^2 = 1000 \times 0.0113\,(35+8)^2 = 12305.7$ N

Work done $= F_x \times u = 12305.7 \times 8 = 98445.6$ N

Force exerted by a jet on a fixed inclined plate :

Let a jet of water coming out from the nozzle strikes an inclined flat plate as shown in fig.

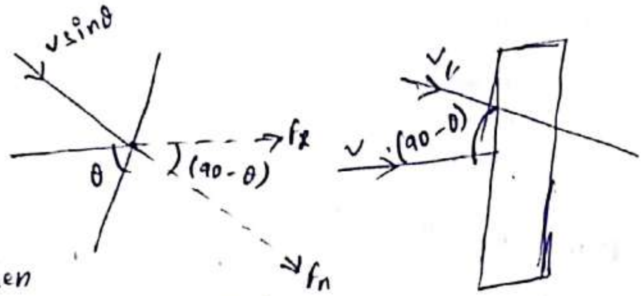


Let v = velocity of jet in the direction of x

θ = Angle b/w jet & the plate

a = C/s area of jet

Then mass of water per second striking the plate $= \rho \times a \times v$



If the plate is smooth & if it is assumed that there is a loss of energy due to impact of jet, then jet will move over the plate after striking with a velocity equal to initial velocity i.e., with a velocity v.

Let us find the force exerted by the jet on the plate in the direction normal to the plate.

Let this force is represented by F_n.

Then F_n = Mass of jet striking per second [Initial velocity − Final velocity]

$$F_n = \rho a v\,(v\sin\theta - 0)$$

$$F_n = \rho a v\,(v\sin\theta)$$

This force can be resolved into two components : One in the direction of jet & other ⊥ to the direction of flow. Then we have

$$F_x = F_n(\cos(90-\theta))$$

$$= F_n \sin\theta$$

$$= \rho a v\,(v\sin\theta)\cdot\sin\theta$$

$$= \rho a v^2 \sin\theta \cdot \sin\theta$$

$$\boxed{F_x = \rho a v^2 \sin^2\theta}$$

$F_y = F_n \sin(90-\theta)$

$= F_n \cos\theta$

$$\boxed{F_y = \rho a v^2 \sin\theta \cdot \cos\theta}$$

Force acting tangentially to the direction of jet $= 0$

∴ Work done per second $=$ Force × Distance

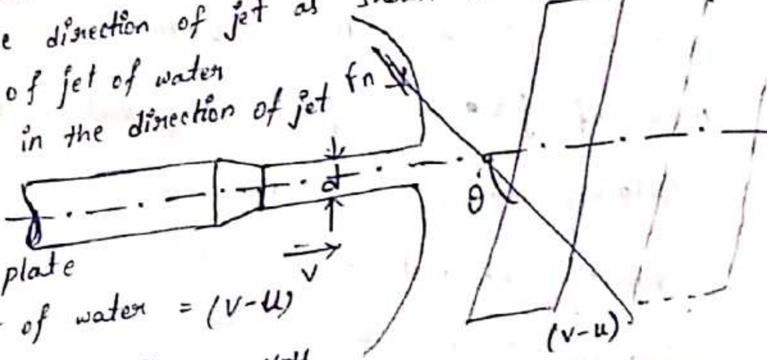
$= \rho a v^2 \sin\theta \times 0$

$$\boxed{W.D/\text{sec} = 0}$$

Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

Force exerted by a jet on a moving inclined plate:

Let a jet of water strikes an inclined plate which is moving with a uniform velocity in the direction of jet as shown in fig.



Let V = Absolute velocity of jet of water

u = velocity of plate in the direction of jet

a = c/s area of jet

θ = Angle b/w jet & plate

Relative velocity of jet of water $= (V-u)$

∴ The velocity with which jet strikes $= V-u$

Mass of water striking /sec $= \rho a(V-u)$

If the plate is smooth & loss of energy due to impact of jet is assumed zero, the jet of water will leave the inclined plate with a velocity $= (V-u)$.

Force acting normal to the plate,

F_n = mass of jet striking per second [Initial velocity − Final velocity]

$= \rho a(V-u)\left[(V-u)\sin\theta - 0\right]$

$F_n = \rho a(V-u)^2 \sin\theta$

This normal force, F_n is resolved into two components namely F_x & F_y in the direction of flow & ⊥ to the direction of flow.

$F_x = F_n\left[\cos(90-\theta)\right]$

$= \rho a(V-u)^2 \sin\theta \cdot \sin\theta$

$$\boxed{F_x = \rho a (V-u)^2 \sin^2\theta}$$

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$F_y = F_n \cos\theta$

$$\boxed{F_y = \rho a (v-u)^2 \sin\theta \cdot \cos\theta}$$

Work done per second $= \rho a (v-u)^2 \sin^2\theta \times u$

① A jet of water of dia 10cm strikes a flat plate normally with a velocity of 15 m/sec. The plate is moving with a velocity of 6m/sec in the direction of jet & away from the jet. Find: i) the force exerted by the jet on the plate
ii) Work done by the jet on the plate per second. $\theta = 30°$

Sol: Given Data:

Dia of jet $d = 10cm = 0.1 m$

Area of jet $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.1)^2 = 0.0078 m^2$

Velocity of jet $v = 15 m/sec$

Velocity of plate $u = 6 m/sec$

Angle b/w jet & plate, $\theta = 30°$

Inclined Fixed
" Moving
Straight Fixed

i) Force exerted by the jet on the plate,

$F_x = \rho a (v-u)^2 \sin^2\theta = 1000 \times 0.0078 (15-6)^2 \sin^2(30°) = 157.95$

ii) Work done /sec $= \rho a (v-u)^2 \sin^2\theta \times u$

$= 1000 \times 0.0078 (15-6)^2 \sin^2(30) \times 6$

$= 947.7$

② A 7.5 cm dia having a velocity of 30m/sec strikes a flat plate, the normal of which is inclined at 45° to the axis of jet. Find normal pressure on plate i) when plate is stationary ii) the normal pressure when plate is moving with velocity of 15m/sec away from jet & also determine the power & efficiency of the jet when plate is moving.

Sol: Given Data: Dia. of jet $= 7.5cm = 0.075 m$

Velocity of jet $v = 30 m/sec$

Velocity of plate $u = 15 m/sec$

Angle, $\theta = 45°$

$= 1000 \times \frac{\pi}{4}(0.075)^2 \times (30)^2 [\sin(45^\circ)]$

$= 2810.96\ N$

$= 15\ m/s$

$= \rho A (v-u)^2 \sin\theta$

$= 1000 \times \frac{\pi}{4}(0.075)^2 (30-15)^2 [\sin(45^\circ)]$

$= 70252.16\ N$

$= \rho A (v-u)^2 \sin\theta * u$

$= 70252.16 \times 15$

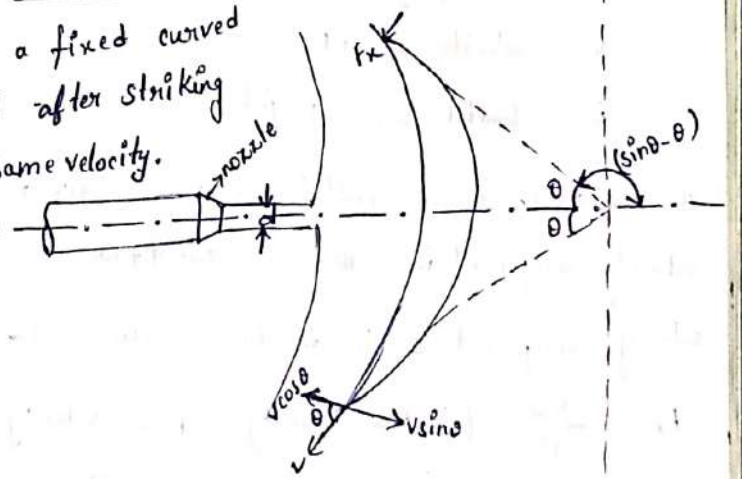
$= 7449.116\ N\text{-}m/sec$

ver $= 7.449$ Watts

ciency $\eta = \frac{\text{Work done}}{KE} = \frac{7449.116}{\frac{1}{2} \times \rho A(v-u)^2} = \frac{7449.116}{\frac{1}{2} \times 1000 \times \frac{\pi}{4}(0.075)^2(30-15)^2} = 0.999$

exerted by a jet on a fixed curved plate :

a jet of water strike a fixed curved at the centre, the jet after striking plate comes out with the same velocity. the plate is smooth & there o loss of energy.



, d = Diameter of jet

a = Area of jet

v = Velocity of jet

θ = Outlet angle of jet

velocity at the outlet can be resolved into two components.

ity component in the direction of flow $= -v\cos\theta$

ity component $\perp$ to the direction of flow $= v\sin\theta$

$F_x = \frac{\text{Mass}}{\text{Time}}$ [Initial Velocity - Final velocity]

$= \rho a v [v - (-v\cos\theta)]$

$= \rho a v [v + v\cos\theta]$

$\boxed{F_x = \rho a v^2 [1+\cos\theta]}$

$y = \rho a v (0 - v\sin\theta) \Rightarrow F_y = \boxed{\rho a v^2 \sin\theta}$

Efficiency , $\eta = 0$

Work done /sec $= 0$

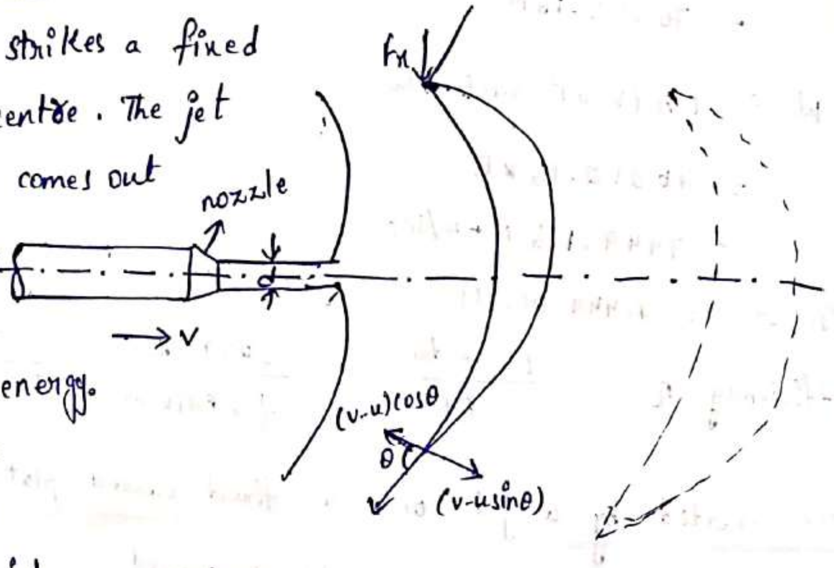
$$\tan\theta = \frac{F_y}{F_x}$$

$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]$$

Resultant force , $F_R = \sqrt{F_x^2 + F_y^2}$

Force exerted by a jet on moving curved plate :

Let a jet of water strikes a fixed curved plate at the centre. The jet after striking the plate comes out with the same velocity if the place is smooth & there is no loss of energy.



Let , d = dia of jet

a = area of jet

v = velocity of jet

θ = Outlet angle of jet.

The velocity at the outlet can be resolved into two components.

velocity component in the direction of flow $= -(v-u)\cos\theta$

velocity component $\perp$ to direction of flow $= (v-u)\sin\theta$.

$$F_x = \frac{\text{Mass}}{\text{Time}}\left[\text{Initial velocity} - \text{final velocity}\right]$$

$$= \rho a(v-u)\left[(v-u) - (v-u)\cos\theta\right]$$

$$= \rho a(v-u)^2(1+\cos\theta)$$

$$F_y = \rho a(v-u)\left[0 - (v-u)\sin\theta\right]$$

$$= -\rho a(v-u)^2\sin\theta$$

Work done = force × distance $= \rho a(v-u)^2(1+\cos\theta)\,u$

$$\text{Efficiency}, \eta = \frac{\text{Work done per sec}}{K.E} = \frac{\rho a(v-u)^2(1+\cos\theta)u}{mv^2}$$

Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips :

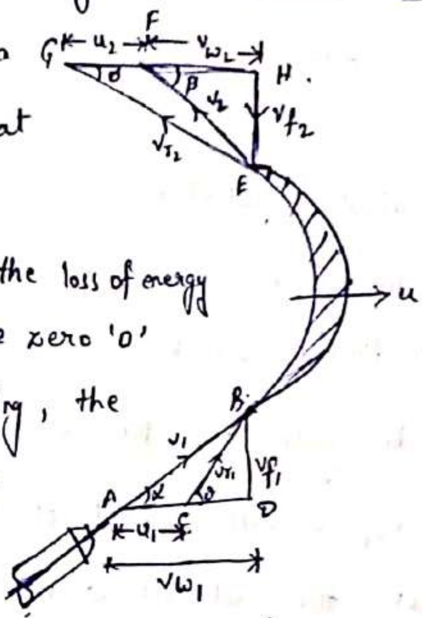


Fig shows a jet of water striking a moving curved plate tangentially at at one of the tips.

As the jet strikes tangentially the loss of energy due to impact of jet will be zero '0'

In this case, the plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet w.r.to the plate. Also as the plate is moving in different direction of the jet, the relative velocity @ inlet will be equal to the vector difference of the velocity of jet & plate at inlet.

Let, V_1 = Velocity of jet @ inlet

u_1 = velocity of plate @ inlet

V_{r_1} = Rel. Vel of jet & plate @ inlet

α = Angle b/w the direction of jet & direction of motion of the plate, also called guide blade angle.

θ = Angle made by Rel.vel with the direction of motion @ inlet.

V_{w_1} & V_{f_1} = The components of velocity of jet V_1 in the direction of motion & $\perp$ to the direction of motion of plate respectively

V_{w_1} = Vel of whirl @ inlet

V_{f_1} = Vel of flow @ inlet

V_2 = Vel of jet @ outlet

u_2 = Vel of plate @ outlet

V_{r_2} = Relative velocity of the jet wrto plate @ outlet

β = Angle made by velocity V_2 with the direction of motion of plate @ outlet

ϕ = Angle made by Rel.vel V_{r_2} with the direction of motion of ~~wave~~ ~~inlet~~ vane @ outlet

V_{w_2} = Velocity of whirl @ outlet

V_{f_2} = velocity of flow @ outlet

Case-i : Velocity Triangle @ Inlet :

Let ABD is the vel. Δ formed by jet of water @ inlet

$BD = V_{f_1}$ $AD = V_{w_1}$

$\angle BCD = \theta$

Case-ii : Velocity Triangle @ Outlet :

If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero.

The water will be gliding over the surface of the vane if the rel. velocity $= V_{r_1}$ & will come out of the vane with a rel. vel V_{r_2}

This means that the rel. vel @ outlet is $V_{r_2} = V_{r_1}$

Draw EG to the tangential direction of the plate @ outlet & cut $EG = V_{r_2}$

From G, Draw a line GF & it is equal to u_2.

$EH = V_{f_2}$; $FH = V_{w_2}$

ϕ = Angle @ outlet

β = Angle made by V_2 with the direction of motion of plate at outlet.

If the plate & smooth & is having velocity in the direction of motion @ inlet & outlet equal then we've $u_1 = u_2 = u$ & $V_{r_1} = V_{r_2}$

Now Mass of water striking the plate per second $= \rho a V_{r_1}$

Force exerted by the jet in the direction of motion, F_x = Mass of water striking per second [Initial vel − Final vel]

But initial vel with which jet strikes the plate $= V_{r_1}$

The component of this velocity in the direction of motion $= V_{r_1} \cos\theta$

$$\cos\theta = \frac{(V_{w_1} - u_1)}{V_{r_1}}$$

$$V_{r_1} \cos\theta = V_{w_1} - u_1$$

Similarly the component of the rel velocity @ outlet in the direction

... motion is $\cos\phi = \frac{\quad}{V_{r_2}}$ $\Rightarrow V_{r_2}\cos\theta = -(u_2 + V_{w_2})$

... sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Sub above eqs in F_x, then we get

$$F_x = \rho a V_{r_1}\left[(V_{w_1} - u_1) - (-(u_2 + V_{w_2}))\right]$$

$$= \rho a V_{r_1}\left[V_{w_1} - \cancel{u_1} + \cancel{u_2} + V_{w_2}\right]$$

$$= \rho a V_{r_1}\left[V_{w_1} + V_{w_2}\right]$$

The above eq is true only when angle β is an acute angle.

If $\beta = 90^\circ$, V_{w_2} should be zero.

$$\therefore F_x = \rho a V_{r_1}(V_{w_1})$$

If β is an obtuse angle, then the expression for F_x is

$$F_x = \rho a V_{r_1}\left[V_{w_1} - V_{w_2}\right]$$

$\therefore$ The general eq for $F_x = \rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]$

Work done per sec on the plate by the jet = force × distance per sec in the direction of force.

$$\text{Work done} = F_x \times u.$$

$$= \rho a V_{r_1}(V_{w_1} \pm V_{w_2})\,u$$

Work done per sec per unit wt of fluid striking per second

$$\text{Work done per sec} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]u}{\text{Weight of fluid striking/sec}}$$

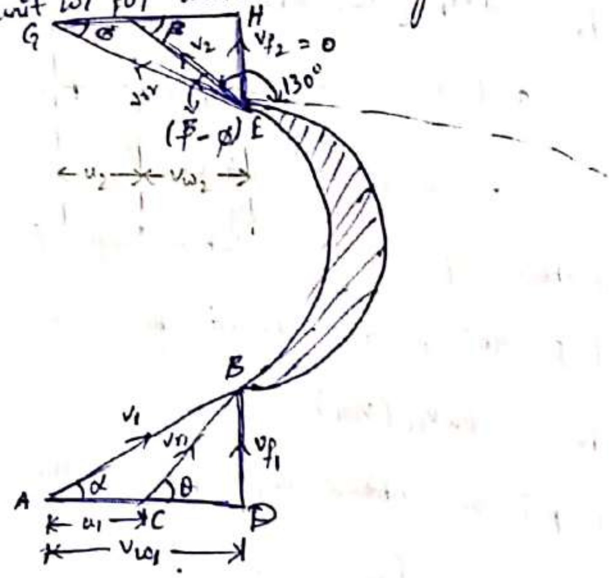
$$= \frac{\cancel{\rho a V_{r_1}}\left[V_{w_1} \pm V_{w_2}\right]u}{g \times \cancel{\rho a V_{r_1}}}$$

$$= \frac{1}{g}\left[V_{w_1} \pm V_{w_2}\right] \times u$$

<u>**Efficiency of jet :-**</u>

$$\eta = \frac{\text{w.D/sec}}{KE} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\frac{1}{2} m V^2} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]u}{\frac{1}{2}\left[\rho a V_1\right] V_1^{\,2}}$$

① A jet of water having a velocity of 20m/sec strikes a curved vane which is moving with a velocity 10 m/sec. The jet makes an angle of 20° in the direction of motion of vane @ inlet & leaves @ an angle of 130° to the motion of vane at outlet. Calculate vane angles, so that the water enters & leaves the vanes without shock. Work done/sec per unit wt of water striking the vane per sec.



Sol: Given Data :

velocity of jet $v_1 = 20 m/s$

velocity of vane $u_1 = 10 m/s$

Angle made by jet of vane @ inlet, $\alpha = 20°$

Angle made by leaving jet with the direction $= 130°$

$\beta = 180° - 130° = 50°$

$u_1 = u_2 = 10 m/s$

$v_{r_1} = v_{r_2}$.

Vane Angles :

From ΔABD

$$\tan\theta = \frac{BD}{CD}$$

$$\tan\theta = \frac{v_{f_1}}{v_{w_1} - u_1}$$

where, $v_{f_1} = v_1 \sin\alpha = 20 \times \sin 20° = 6.84$

$v_{w_1} = v_1 \cos\alpha = 20 \times \cos 20° = 18.79\ m/s$

$$\tan\theta = \frac{v_{f_1}}{v_{w_1} - u_1} = \frac{6.84}{18.79 - 10} = 0.778.$$

$\theta = \tan^{-1}(0.778)$

$\therefore \theta = 37°$

In ΔABC, $\sin\theta = \dfrac{V_{f_1}}{V_{r_1}}$

$$V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37^\circ} = 11.36$$

$$V_{r_1} = V_{r_2} = 11.36 \text{ m/sec}$$

In ΔEFG applying sine rule, we have

$$\frac{V_{r_2}}{\sin(180^\circ - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

$$\frac{11.36}{\sin\beta} = \frac{u_2}{\sin(\beta - \phi)}$$

$$\frac{11.36}{\sin 50^\circ} = \frac{u_2}{\sin(50 - \phi)}$$

$$u_2 = 14.82 \sin(50^\circ - \phi)$$

$$\sin(50 - \phi) = \frac{10}{14.82}$$

$$50 - \phi = \sin^{-1}(0.674)$$

$$-\phi = \sin^{-1}(0.674) - 50$$

$$+\phi = \cancel{-}\, 7^\circ 37'$$

$$\phi = 7^\circ 37'$$

Work done per sec per unit weight of water striking the vane per second $= \dfrac{1}{g}\left[V_{w_1} \pm V_{w_2}\right] u$

Note: +ve sign is to be taken because $\beta = 50^\circ$ [$\because$ acute angle]

$$V_{w_1} = 18.79 \text{ m/s}$$

$$\cos\phi = \frac{V_{w_2} + u_2}{V_{r_2}}$$

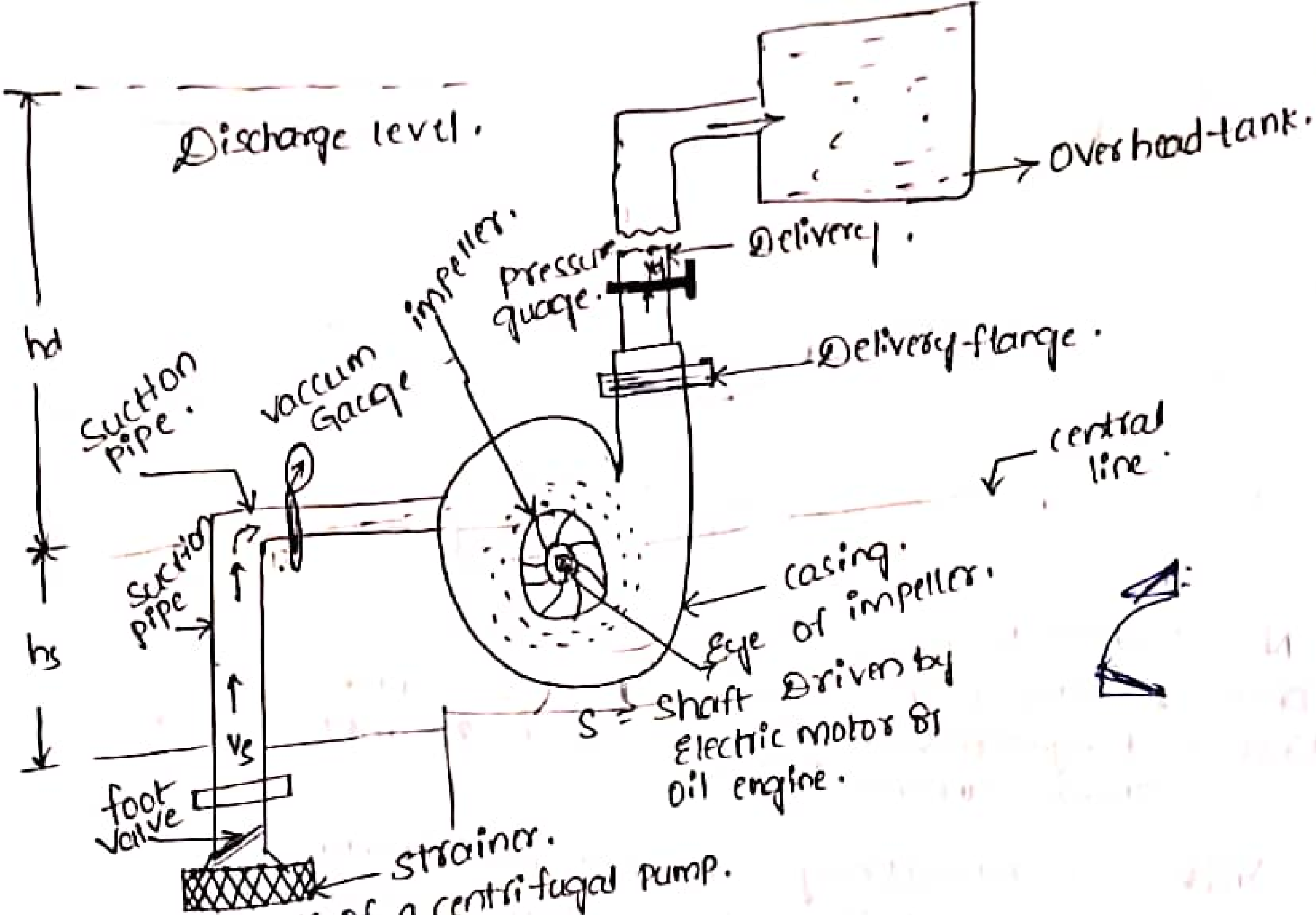
$$V_{r_2}\cos\phi - u_2 = V_{w_2}$$

$$V_{w_2} = 11.36\cos(7^\circ 37') - 10 = 1.26 \text{ m/s}$$

$$WD = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u$$

$$= \frac{1}{9.81}\left[18.79 + 1.26\right]$$

$$= 20.24 \text{ m/sec}$$



Main parts of a centrifugal pump.

Working of centrifugal pump:-

The working of a centrifugal pumps starts with priming is the process of filling the suction pipe, casing and the delivery pipe upto the delivery valve by the liquid which is to be pumped. Priming is done to drive out the air pockets present.

* Once priming is done, keeping the delivery valve still closed the electric motor is started which rotates the impeller. The centrifugal force induced due to forced vortex increases the pressure energy of the liquid. As long as the delivery valve is closed, the liquid gets churned inside the casing and gets more energy.

Once the delivery valve is opened, the liquid rushes into the delivery pipe. This empties the casing of the centrifugal pump creating a partial vaccum at the centre of the pump, there by the making the continuous process of pumping of the liquid.

Workdone by the centrifugal pump:-

Incase of the centrifugal pump work is done by the impeller on the water. The expression for the working workdone by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as explained for turbines.

The water enters the impeller at its centre (radially i.e $\alpha = 90^\circ$ and $V_{w1} = 0$) and leaves at its outer periphery. For drawing the velocity triangles, the same notations are used as that wave earlier.

Definition of Heads:-

1. Suction head (h_s):- It is the vertical height of the centre line of the centrifugal pump above the water surface in the pump from which water is to be lifted. It is denoted by h_s.

2. Delivery head:- The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by h_d.

3. Static head (H_s):- The sum of suction head and delivery head is known as static head. This is represented by H_s and is written as $H_s = h_s + h_d$.

4. Manometric head (H_m):- The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the loss of impeller to the water head in pump.

H_m = Head imparted by the impeller to the water - Loss of head.

$= \frac{V_{w_2} U_2}{g}$ - Loss of head

$= \frac{V_{w_2} U_2}{g}$ (if loss of head is zero).

centrifugal pump:- ... transmitted from the shaft

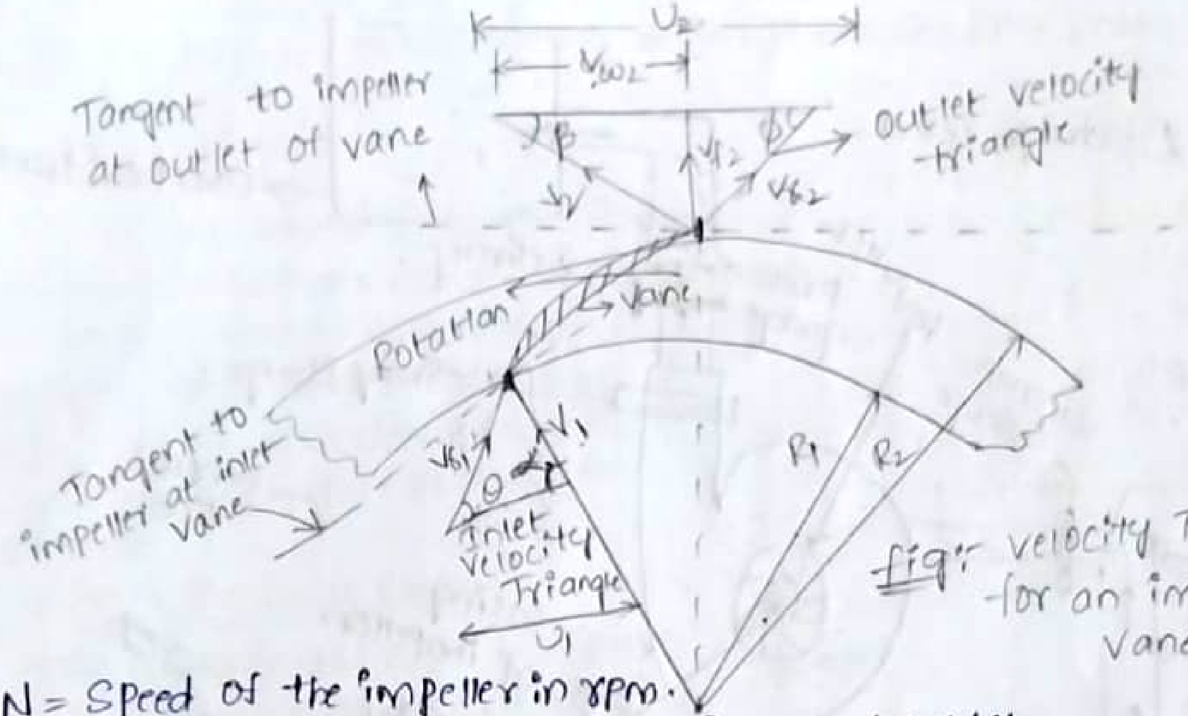


fig: velocity Triangle for an impeller vane

N = Speed of the impeller in rpm.

D_1 & D_2 = Diameter of the impeller at inlet and outlet.

U_1 & U_2 = Tangential velocity of impeller at the inlet & outlet.

$= \frac{\pi D_1 N}{60}$ & $\frac{\pi D_2 N}{60}$.

V_1 & V_2 = Absolute velocity of liquid at inlet & outlet.

V_{r_1} & V_{r_2} = Relative velocity of liquid at inlet & outlet

V_{w_1} & V_{w_2} = Velocity of whirl at inlet and outlet

V_{f_1} & V_{f_2} = velocity of flow at inlet & outlet.

α = Jet angle at inlet it is the angle made by absolute velocity (V_1) at inlet with the direction of motion of vane.

θ = vane angle at inlet, it is the angle made by the relative velocity (V_{r_1}) at inlet with the direction of motion of vone.

β & ϕ are the corresponding values at outlet A centrifugal pump is just the reverse of a radially inward flow reaction turbine. Thus,

workdone by centrifugal pump (or impeller) per second = -workdone by radial inward flow reaction turbine per second.

$= -\rho Q (V_{w_1} U_1 - V_{w_2} U_2) = \rho Q (V_{w_2} U_2 - V_{w_1} U_1)$.

$\frac{\rho g Q}{g} \quad \frac{w Q}{g}$

$= -\rho a V_1 [V_{w_1} U_1 - V_{w_2} U_2]$

$= \frac{wQ}{g} [V_{w_2} U_2 - V_{w_1} U_1] = \frac{wQ}{g} \times V_{w_2} U_2 = \rho Q (V_{w_2} U_2)$ $[\because V_{w_1} = 0]$.

work done by the impeller on the water per unit weight of water.

$= \frac{1}{g} [V_{w_2} U_2 - V_{w_1} U_1] = \frac{1}{g} [V_{w_2} U_2]$.

Power output of the pump $= wQH_m = \frac{wQ V_{w_2} U_2}{g}$ watts.

Discharge = Area × velocity [Q (or) volume of water].

$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$.